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OF THE

ROYAL ASTRONOMICAL SOCIETY.

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H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

Hugh Cameron Campbell, Science Department, Surgeons' Hall, Edinburgh,

and

The Rev. Arthur Mackreth Deane, M.A., Canon of Chichester, Ferring Vicarage, Worthing, Sussex,

were balloted for and duly elected Fellows of the Society.

Benjamin Baillaud, Director of the Observatory, Paris;

Carl Ludwig Wilhelm Charlier, Director of the Observatory, Lund, Sweden;

Edwin Brant Frost, Director of the Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.;

Johann Georg Hagen, S.J., Director of the Vatican Observatory, Rome; and

Johannes Franz Hartmann, Astrophysical Observatory, Potsdam, Germany,

were balloted for and duly elected Associates of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended:—

The Rev. Thomas Nicklin, M.A., Assistant Master, Rossall School, Bescot, Rossall Beach, Fleetwood, Lancashire (proposed by S. A. Saunder).

Eighty-one presents were announced as having been received since the last meeting, including amongst others:—Cape Catalogue of 1680 stars for equinox 1900·0, presented by the Royal Observa-

tory, Cape of Good Hope; G. E. Hale, The Study of Stellar Evolution, presented by the author; five spectroheliographs (enlargements), showing hydrogen and calcium flocculi, presented by the Mount Wilson Observatory; lithograph from a drawing of Donati's Comet, by Miss Charlotte S. Cooper, Markree, presented by Miss Cooper; 16 charts of the Astrographic Chart of the heavens, presented by the Royal Observatory, Greenwich.

Considerations on the Form and Arrangement of New Tables of the Moon. By Simon Newcomb.

Hansen's Tables of the Moon, with some patching up, have now been in use in the standard astronomical ephemerides for a full half century. With every passing year the necessity of replacing them by improved ones becomes more urgent. My own investigations on the Moon's motion have always been carried on with the view of ultimately constructing new tables, while experience in the use of Hansen's tables has from time to time formed a basis for a study of the relative merits of various forms of the lunar theory for the purpose of tabulation. But, as the years have passed, it has become increasingly apparent that I must leave to other hands the execution of the desired work. I therefore venture to summarise the suggestions which I have to make on the whole subject.

§ 1. *General Form of Tables.*

All my experience with Hansen's tables has led me to the conclusion that if the problem were only that of enabling an ephemeris of the Moon to be computed with the use of the fewest figures, it would scarcely be possible to improve on Hansen's arguments and system of tabulation. The Hansenian perturbations of the mean anomaly are more rapidly convergent than those of any co-ordinate that can be used, and therefore require fewest tables. The expression of the fundamental argument in terms of the time may also have a certain advantage, though this is a point on which I feel less confident, because the best unit is not a decimal of a day.

But the mental labour to be performed is not measured merely by the number of figures taken out and written down. A "straight-ahead" computation has a decided advantage over one in which it is necessary to form numbers from preceding tables, or to introduce tables of logarithms, as we must when we pass from Hansen's fundamental argument to the true longitude. This remark is especially applicable to the reduction to the ecliptic. The mental labour of forming arguments from numbers already used and of combining results from various tables is intensified by the care and attention which then have to be bestowed, and which it is desirable to render unnecessary.

The desired uniformity in proceeding would be attained in the highest degree by tabulating the perturbations of the ecliptic longitude and latitude in the form to which they are reduced by Brown, using a uniform system of arguments for the tables. But the number of tables then requisite would be so great that it is doubtful whether they could be put into a single volume of reasonable size, say that of Hansen's tables as they now are. The most troublesome and important terms would be those arising from the reduction to the ecliptic. It may be added that I have never found Hansen's system of constructing the double-entry tables for intervals of $0^d.25$ to offer a great advantage over the use of 12-hour intervals, except when an isolated place is to be computed.

The modification of the plan which I have to suggest will best be seen by starting with Hansen's fundamental argument. After it is formed, two steps are required—the reduction from the inequality of mean longitude to that of true longitude in orbit, and the reduction to the ecliptic. When the reductions are made in the general formulæ, tabulated, as they will be if ecliptic co-ordinates are tabulated, each of these steps adds a number of additional inequalities, but I think those arising from the second reduction exceed in number and magnitude those arising from the first. However this may be, it is certain that the tabulation of the true longitude will require much more voluminous tables than will that of the Hansenian mean longitude.

What I would suggest is, therefore, the tabulation of the longitude in orbit, that co-ordinate being defined as the distance from the mean node to the circle of latitude passing through the Moon perpendicular to the ecliptic. More geometric symmetry would be attained by taking as the fundamental plane the mean moving orbit of the Moon, and referring the longitude and latitude to that plane. But this would require a multiplication of the quantities by certain factors in order to find the true longitude. If the principal term of the latitude is computed with the true argument of latitude, the inequalities are fewer and smaller than if the mean argument is used.

§ 2. *Unit of Tabular Longitude.*

In the case of the longitude, the next question is that of the unit to be chosen. It is so important to avoid the use of larger numbers than is necessary that this choice requires careful consideration. The tabulated unit of Hansen's fundamental argument is approximately $0''.005$. The smallness of this quantity makes the interpolation laborious without really adding materially to the accuracy of the result. To devise the best unit, we must first consider the degree of precision to be reached by some arbitrarily chosen unit, say $0''.01$. Here some statements relating to the accumulation of small accidental errors become necessary.

§ 3. *Errors accumulating from omitted decimals.*

The theory of errors accumulating from omitted decimals is not usually developed in a way that seems to me entirely satisfactory. I therefore begin by stating some principles, definitions, and results pertaining to the subject.

1. I use the term *probable value* of a doubtful quantity in the sense introduced by Poincaré, which is—

If $$v_1, v_2, \ldots . v_n$$

be all the essentially different possible values of a quantity v, and if

$$p_1, p_2, \ldots . p_n$$

be the several probabilities of these values, then the probable value of v is that given by the equation

$$v = p_1 v_1 + p_2 v_2 + \ldots . + p_n v_n .$$

That is, the probable value is the mean of all the possible values, each weighted according to its probability.

2. I deal, in the case of each quantity, with the probable value of the square of its deviation from the truth. The square root of this square is generally designated in astronomy as the *mean error*. Pearson has introduced the somewhat more appropriate term of "standard deviation," which I abbreviate to S.D. We may therefore define this as the square root of the probable value of the square of the deviation.

3. Let us consider the sum S of any number n of independent quantities, $q_1, q_2, q_3, \ldots . q_n$.

Let these q's be uncertain by the several standard deviations

$$\epsilon_1, \epsilon_2, \epsilon_3 \ldots . \epsilon_n .$$

If we put ϵ for the S.D. of S we shall then have

$$\epsilon^2 = \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + \ldots . + \epsilon_n^2 .$$

The advantage of this equation is that it is independent of the law of probability of an error as a function of its magnitude, which is not in all cases the normal law. For example, the error arising from the omission of a periodic term has a well-defined absolute maximum equal to the coefficient of the term. Moreover, its maximum value is more probable than any other. The standard deviation arising from the omission of a periodic term $a \sin mt$ is

$$\epsilon = \pm \frac{a}{\sqrt{2}} .$$

An examination of the coefficients of longitude found by Brown and Hill shows that there are about 300 periodic terms whose

coefficients lie between the limits 0″·01 and 0″·001. I take this lower limit as that beyond which it is unnecessary to go. In estimating the S.D. I take 0″·01 as the unit. An approximate estimate of the distribution of these coefficients in magnitude shows that the square of the S.D. produced by omitting them is, with sufficient exactness,

$$\epsilon^2 = 28 .$$

There remain, by a rough estimate, about 300 terms exceeding the limit 0″·01, nearly all of which I assume will be tabulated. I also assume that the individual numbers of these tables will be formed by carrying each number to 0″·001, and that the last decimal will be dropped in the final tabulated number, which will be given only to 0″·01. Making due allowance for all imperfections, I find that the standard deviation of a number interpolated from a single table thus formed will be ±0·26 if the number is written down to one decimal beyond that of the tables. But if the number is, as usual, only written to the tabular number of decimals, the deviation will be ±0·39.

I assume that not more than 120 tabular numbers will be added to form the longitude. The contribution to $(\text{S.D.})^2$ arising from the summation of these numbers is

$$120 \times 0{\cdot}39^2 = 18{\cdot}2 .$$

The imperfections of the arguments will also have their influence. The deviation produced by them may be reduced by giving each argument to one decimal beyond that required by the condition that the error arising from a unit error of the argument shall always be less than that of the tabular unit. But, without going beyond this rule, the effect of the errors of the argument will not exceed that arising from the addition of thirty more tabular numbers. We may thus have—

$$(\text{S.D.})^2 \text{ from errors of arguments} = 4{\cdot}5.$$

In some cases the effect of the imperfections of the correction for the second differences may add to the S.D. I think that, with a little skill and attention on the part of the computer, this S.D. need not exceed ±0·2, giving 4·8 for its entire contribution to $(\text{S.D.})^2$.

Summing up all the sources of accidental deviation of the tabular results from theory we have—

From the omission of small terms . .	$(\text{S.D.})^2 = 28$
,, ,, tabular decimals .	,, 18
,, errors of arguments . . .	,, 5
,, ,, of second diff. . .	,, 5
In all	$(\text{S.D.})^2 = 56$

And for the sum, S.D. = 7·5.

We shall therefore have, in the case of each computed longitude, a S.D. of $\pm 0''.075$ and a probable error of $0''.05$.

§ 4. *Degree of precision required.*

Let us compare this with the degree of precision required in a comparison with observation. My experience in the use and examination of lunar observations leads me to the conclusion that no single observation of any sort can be practically made without a mean error exceeding $\pm 0''.5$, corresponding to a probable error of $\pm 0''.34$. It is desirable that the mean error of a co-ordinate found from the tables should be less than this. But all the results obtained from lunar observations depend upon a great number of observations, which make the accidental errors unimportant in comparison with the systematic ones. The practical advantage of a degree of precision above that just mentioned in the tabular places is very small, and is practically evanescent if reduced below $\pm 0''.4$. This degree of precision will be surpassed by adopting as the unit to be tabulated the 10^{-8} part of the circumference, or $0''.01296$. Of course the unit $0''.01$ would answer our purpose if deemed more convenient in use. But the smaller the numbers used, the easier the computer will find it to avoid small errors, while the S.D. will still fall below $\pm 0''.09$.

There is, however, one point still to be considered in this connection which may modify our conclusions. Granting an S.D. of $\pm 0''.10$ in the individual longitudes, we must expect that in the course of a year there may be three or four of the 730 tabulated longitudes in error by three times the S.D., and possibly one of four times this amount. But every error approaching such a magnitude as this will be detected by differencing the 12-hour ephemeris. A legitimate proceeding will then be to smooth off the ephemeris by such small corrections as shall reduce the higher, say the fifth or sixth, differences to a sufficiently smooth series. Each corrected tabular result may then be regarded as the mean of two or more neighbouring quantities, and the maximum error of the ephemeris will be reduced nearly to the mean S.D. In a word, we may fairly count on having an ephemeris in which all the errors exceeding some limit between $0''.12$ and $0''.30$ will be eliminated. This limit is still within the errors of the best observations, and the cases in which it is approached will be rare.

§ 5. *Reduction to sexagesimal units.*

The proposed units will require the reduction of the final longitudes to degrees, minutes, and seconds. The tables necessary for this purpose will perhaps fill four pages, and the computation will be equivalent to the entry of three additional tables.

If the unit $0''.01$ is deemed preferable, its use will still require some study. It was adopted in Peirce's Tables of the Moon,

published in 1853. The tabular numbers were there expressed in degrees and seconds, minutes being ignored. I found the use of this system cumbrous, and should prefer to use seconds pure and simple, subtracting 1,296,000″ or its multiples when necessary. The proposed circumferential unit does away with this subtraction. Although it is a little easier to change the degrees into seconds and minutes than it is to change the circumferential unit, I still think the advantage to lie with the latter.

It may be of interest in this connection to note that if we should base the unit on the degree, tabulating to 0°·00001, the S.D. of the individual longitudes would still be only ±0″·18, and we might be fairly confident that no error exceeding 0″·4 would remain in a smoothed-off annual ephemeris as often as once a year.

Of course all this presupposes that the computer is always careful never to make a greater error than 0·5 in interpolating and writing down his number. The question may arise whether it is not well to allow him a margin of one or two units, by adopting smaller units. My answer is that the labour of handling large numbers involves more mental strain than that required in the accurate handling of small numbers, and that the assigned standard of precision will be more easily reached by the careful use of the smaller numbers than by the careless use of the larger ones.

§ 6. *Epochs and Arguments.*

For the practical work of computing places of the Moon for given dates, I do not think that any system more convenient than the usual one can be devised. The Hansenian form, in which the Gregorian and Julian calendars are used, is the most convenient of all. But it is always desirable to give the tables such a form that the relation between the tabular numbers and the original elements shall be easily examined, and corrections to the theory readily applied. This suggests a slight sacrifice of ease in computing an isolated position, or an ephemeris, to the requirements of the theoretical investigator.

Simplicity in the other direction is reached by the use of days of the Julian period. This was first employed, I believe, by Peirce, and is now extensively used in astronomy, especially in Oppolzer's works relating to eclipses. In using this system, a first and easy step is the reduction of the ordinary calendar date to days of the Julian period. Then absolute uniformity is reached in the construction and use of the tables.

The principal immediate drawback of this system is that, if used unmodified, the period of 1000 days must take the place of the year. The formation of the arguments of short period is then inconvenient. Many of the lunar arguments have periods not differing much from a month. From 12 to 15 multiples then suffice when the year is used, but with the period of 1000 days the number of multiples to be tabulated and subtracted will frequently be between 30 and 40, and some times more. Of course

this difficulty can be lessened by taking 500 days instead of 1000 as the second unit. But this will detract from simplicity of form.

A yet more serious drawback to the theoretical investigator is that the fundamental epochs usually adopted in astronomy, and for which the elements must be found, do not correspond to any power of 10 in the days of the Julian period. A complete transformation of the elements is therefore required to form the numbers on which the tables are based. If, therefore, multiples of 500 or 1000 days are used instead of years, I should prefer to count them back from 1900·0, thus gaining all the advantages of the Julian period without any other disadvantage than that of non-correspondence with the eclipse and other tables of Oppolzer.

The reduction of such a system to the ordinary calendar may be made a very simple matter. It seems to me, therefore, that the maximum of advantage will be reached by giving the fundamental arguments for cycles and periods based on multiples of 500 days before and after the fundamental epoch 1900 Jan. 0.

Probably the most convenient fundamental quantities to tabulate will be the longitude of the node, and the mean distances of the Moon and of its perigee from the node, all expressed in circumferential units. Then, whatever form the tables may be thrown into, we shall have the nearest approach to a simple, straight-ahead computation.

Finally, a serious problem is that of summing perhaps 100 periodic terms with coefficients not differing greatly from 0″·01. I have devised a machine for this purpose, the description of which must form the subject of another publication.

An Example of Professor Karl Pearson's Calculation of Correlation in the case of the Periodic Inequalities of Long-period Variables. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The following note is written with a twofold purpose. Firstly, it is hoped that an indication of some value has been obtained with regard to the features of "long-period" variability; and secondly, the opportunity is taken to write out in full a simple example of the calculation of "correlation" between quantities by the methods of Professor Karl Pearson.

In the *M.N.* for March last (p. 416) Professor Pearson himself gave an admirable summary of methods; but he naturally did not repeat the elementary working which has become so familiar to him, and has been given often before in other connections. There are doubtless many to whom this working is already familiar; but there are certainly many others who do not know it and who might use it if they had an astronomical example readily accessible. In these busy days many people have not the leisure to search for references in scientific literature outside their own subject.

2. In support of my view that there is need for an example, I may cite an illustration shown me by Professor E. C. Pickering (see *Observatory*, March 1905, p. 153).

Suppose we have the following observed values of A and B:—

A . . .	5	6	7	6	7	8	7	8	9
B . . .	6	6	6	7	7	7	8	8	8

First group according to B, in sets of three for which B has the same value.

Then when $B = 6 \quad 7 \quad 8$

Mean $A = 6 \quad 7 \quad 8$

Therefore $A = B.$

Secondly, group according to A.

Then when $A = 5 \quad 6 \quad 7 \quad 8 \quad 9$

Mean $B = 6 \quad 6\frac{1}{2} \quad 7 \quad 7\frac{1}{2} \quad 8$

Therefore $(A - 7) = 2(B - 7).$

Now, neither Professor Pickering nor myself was prepared at that time to deal with this situation by a definite process; and yet this is just an elementary case of the kind which Professor Pearson's methods were devised to meet.

This example is worked out below, and it is shown that the numerical measure of the correlation is

$$r = \cdot 08 \pm \cdot 22$$

or almost nothing at all: so that, in spite of appearances to the contrary, we are not entitled to assume any relationship between A and B. Putting it in another way, one proposed relation $(A = B)$ is as good as another $(A - 7) = 2(B - 7)$.

3. A point of detail may be mentioned here. In much statistical work, a large number of figures are used. Thus we get such statements* as

$$r = \frac{1262 \cdot 51}{2870 \times 1 \cdot 13637 \times 1 \cdot 52135} = \cdot 25445.$$

The probable error of r being about $\pm \cdot 01$, some of these figures are superfluous; and in what follows fewer figures are used. This, however, represents a personal view which is, I find, not generally approved by other workers.

4. The particular example selected for treatment is the discussion of the elements of maximum given by Chandler for long-period variables. The following particulars are taken from

* *Frequency Curves and Correlation*, by W. P. Elderton, p. 119: an excellent little book, from which much is to be learned.

his "Revision of Elements of Third Catalogue" (*Astronomical Journal*, No. 553):—

Star.	Epoch.	Elements of Maximum.	Period.		Inequalities.
R Androm.	2400141	+	410·3 E	+	30 sin (12° E + 90°)
S Cassiop.	2401603	+	610·5 E	+	37 sin (15° E + 59°)

and so on. The mean period of S Cassiop. is 610·5 days; but there is an inequality which displaces the maximum 37 days one way, and 12 periods later (since $180°/15° = 12$) the displacement is 37 days in the other direction. At these times the period is about the mean, but midway between such times it is longer and shorter. In the "Revision" above mentioned, Chandler gives such inequalities for 37 stars, as shown in Table I. We shall for the present only consider the period P, the coefficient C, and the argument A; thus for S Cassiop. $P = 610·5$, $C = 37$, and $A = 15°$. The epoch 59° does not concern us at present.

TABLE I.

Star's Name.	No.	P	C	A	Phase.	M − *m*	αP
R Androm.	112	410	30	12	218	120	−170
S Cassiop.	432	610	37	15	148	280	− 50
S Piscium	434	404	18	10	57	163	− 78
R Arietis	782	187	7	5	292	92	− 3
o Ceti	806	332	18	5	...	125	− 82
R Persei	1222	210	15	8	224	96	− 18
R Aurigæ	1855	459	19	12	320	235	+ 11
R Lyncis	2478	379	14	15	249	186	− 7
R Gemin.	2528	370	35	6	121	121	−128
S Can. Min.	2684	330	20	12	120	164	− 2
R Cancri	2946	362	60	6	341	125	−112
R Carinæ	3418	310	25	9	289	136	− 38
R Leo Min.	3477	371	20	10	11	165	− 41
R Urs. Maj.	3825	302	11	8	63	110	− 82
T Urs. Maj.	4511	257	20	9	203	108	− 42
R Virginis	4521	145	20	2	103	68	− 9
S Urs. Maj.	4557	226	35	5	309	108	− 10
S Boötis	5157	270	60	4	87	132	− 6
R Camelop.	5190	270	65	4	244	142	+ 14
R Boötis	5237	223	9	9	251	102	− 20

TABLE I.—*continued.*

Star's Name.	No.	P	C	A	Phase.	M − *m*	*a*P
S Serpentis	5501	369	116	4°	122°	...	...
S Coronæ	5504	361	8	12	81	120	- 121
R Serpentis	5677	357	35	4	109	151	− 55
R Herculis	5770	318	17	10	29	...	...
W Herculis	5950	280	26	13	288	128	− 24
S Herculis	6044	308	35	9	239	152	− 4
T Herculis	6512	165	10	5	152	79	− 7
R Sagittarii	6905	269	18	10	18	138	+ 7
S Sagittarii	6921	231	15	10	167	102	− 27
R Delphini	7261	284	26	9	266	130	− 24
U Capric	7455	203	20	5	67	...	...
T Aquarii	7468	203	8	7	357	88	− 27
R Vulpec.	7560	137	18	4	127	62	− 13
X Capric.	7577	218	20	10	98	117	+ 16
R Pegasi	8290	378	60	8	71	172	− 34
R Aquarii	8512	387	35	10	97	...	...
R Cassiop.	8600	432	32	9	233	182	− 68

5. The first two columns of Table I. give the star's name and its number on Chandler's system. The third column, P, gives the period in days; C is the coefficient of the periodic inequality in days, and A the coefficient of E within the bracket. The explanation of the last three columns will be given later.

6. Now the example selected to be given in full is that of the relation between P and A. To determine whether there is such a relation, we could, of course, proceed in the ordinary way to group together stars with nearly the same P, and take the mean values of A for them. This would give us a fair indication, but no notion of its probable error. Professor Pearson's procedure is not much longer; it includes the ordinary procedure, and it gives us a definite measure of the probability of the relation being real.

7. We first form a "correlation table" as in Table II.

[TABLE

TABLE II.

Correlation Table for P *and* A.

P in days. \ A =	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	Sums
601 to 650														1 (36)	1
551 to 600														0 (30)	0
501 to 550														0 (24)	0
451 to 500								0	0 (3)	0 (6)	1 (9)	0	0	0 (18)	1
401 to 450								1	1 (2)	0 (4)	1 (6)	0 (8)	0 (10)	0 (12)	3
351 to 400			2 (5)	0 (4)	2 (3)	0 (2)	1 (1)	0	2 (1)	0 (2)	1 (3)	0 (4)	0 (5)	1 (6)	9
301 to 350			0	1	0	0	1	2	1	0	1	0	0	0	6
251 to 300			2 (5)	0 (4)	0 (3)	0 (2)	0 (1)	2	1 (1)	0 (2)	0 (3)	1 (4)	0 (5)	0 (6)	6
201 to 250			0 (10)	2 (8)	0 (6)	1 (4)	1 (2)	1	2 (2)	0 (4)	0 (6)	0 (8)	0 (10)	0 (12)	7
151 to 200			0 (15)	2 (12)	0 (9)	0 (6)	0 (3)								2
101 to 150	1 (28)	0 (24)	1 (20)	0 (16)	0 (12)	0 (8)	0 (4)								2
Sums	1	0	5	5	2	1	3	6	7	0	4	1	0	2	37

A	Sum.	Factor.	ν_1 = Product.	ν_2 = Product.
°				
2	1	−7	−7	49
3	0	−6	0	0
4	5	−5	−25	125
5	5	−4	−20	80
6	2	−3	−6	18
7	1	−2	−2	4
8	3	−1	−3	3
9	6	0	0	0
10	7	+1	+7	7
11	0	+2	0	0
12	4	+3	+12	36
13	1	+4	+4	16
14	0	+5	0	0
15	2	+6	+12	72
Sums	37	...	−28	410
Means (ν_1 and ν_2)			−·84	+11·08

$\sigma_A = \sqrt{\nu_2 - \nu_1^2} = \sqrt{10{\cdot}37} = 3{\cdot}22$

P	Sum.	Factor.	ν_1 = Product.	ν_2 = Product.
125	2	−4	−8	32
175	2	−3	−6	18
225	7	−2	−14	28
275	6	−1	−6	6
325	6	0	0	0
375	9	+1	+9	9
425	3	+2	+6	12
475	1	+3	+3	9
525	0	+4	0	0
575	0	+5	0	0
625	1	+6	+6	36
Sums	37	...	−10	150
Means (ν_1 and ν_2)			−·27	4·05

$\sigma_P = \sqrt{\nu_2 - \nu_1^2} = \sqrt{3{\cdot}98} = 2{\cdot}00$

Products $S'(AP$

	−17 +64
	+104 −9
	= +142
37(−·84)(−·27)	= +9
Difference	= 133

$r = \frac{133}{37 \times 3{\cdot}22 \times 2{\cdot}00} = 0{\cdot}56$

8. This requires little explanation. We divide the periods into groups of 50 days as shown in the first column, and the number of periods in each group is shown in the final column. For the values of A no collection in groups is necessary, successive degrees being shown in the top line, and the number of corresponding cases in the bottom line. In the body of the table are given, in large figures, the number of cases corresponding to a definite degree for each group of periods. The small figures will be referred to presently.

9. The thick lines indicate the groups which contain approximately the mean of each co-ordinate. They are chiefly for convenience in working (to keep the numbers small), and if a wrong group is taken no real harm is done.

10. The figures below the table represent the whole working. We first form ν_1 and ν_2 for each co-ordinate by a process which is tolerably clear to inspection. For A write down the (top line for convenience only—not used in working; and the) bottom line of the table. Follow with a sequence of units, with 0 at the place of the adopted mean. Multiply the two columns; and again multiply the last two columns. We thus get $n\nu_1$ and $n\nu_2$, where n is the number of observations, and ν_1, ν_2 two quantities required. Dividing by n, we get ν_1 and ν_2, and finally

$$\sigma_A = (\nu_2 - \nu_1^2)^{\frac{1}{2}} = 3{\cdot}22 .$$

[If greater accuracy is required, we should subtract also "Sheppard's correction" of $\frac{1}{12}$ from ν_2 before taking the square root.]

11. The same procedure with the columns gives us $\sigma_P = 2{\cdot}00$.

12. We now form the product S(AP). The number in each box is to be multiplied by the little number shown in the corner of the box, which will be seen to be the product xy, where x represents the number of boxes to the right, and y the number up, from the adopted mean. The products in the four main divisions of the table are shown diagrammatically, and from them we get

$$S'(AP) = 142 .$$

From this we subtract $n\nu_1\nu_1^1$, where n is the number of observations, and ν_1, ν_1^1 have been found; and thus

$$S(AP) = S'(AP) - n\nu_1\nu_1^1 = 142 - 9 = 133 .$$

Then the "correlation" r is

$$r = \frac{S(AP)}{n \times \sigma_A \times \sigma_P} = 0{\cdot}56 .$$

13. This is quite a respectable value for r—anything over 0·5 is worth serious attention. But to complete our information we want the probable error of r, which is given by the formula

$$p.e = {\cdot}67(1 - r^2)/\sqrt{n} ,$$

which can be taken from the following small table (Table III.).

For the present case we have $n = 37$, $r = 0{\cdot}56$, and thus probable error $= \pm 0{\cdot}08$.

TABLE III.

Probable error of r.

n \ r	·1	·2	·3	·4	·5	·6	·7	·8	·9
10	·208	·202	·193	·177	·158	·136	·107	·076	·041
20	·148	·143	·137	·125	·112	·096	·076	·054	·029
30	·121	·117	·112	·102	·091	·079	·062	·044	·024
40	·104	·101	·096	·088	·079	·068	·054	·038	·021
50	·093	·090	·086	·079	·070	·061	·048	·034	·018
60	·085	·083	·079	·072	·064	·056	·044	·031	·017
70	·079	·076	·073	·067	·059	·051	·040	·029	·015
80	·074	·072	·068	·063	·056	·048	·038	·027	·015
90	·069	·067	·064	·059	·053	·045	·036	·025	·014
100	·066	·064	·061	·056	·050	·043	·034	·024	·013

14. Coming now to the relation between P and A, the mean value of A is got by adding ν_1 to our adopted mean, *i.e.* it is $9^\circ - 0^\circ{\cdot}84 = 8^\circ{\cdot}16$; and the mean value of P is $325 - 0{\cdot}27 \times 50 = 311{\cdot}5$ days, the unit adopted in the table being 50 days, by which therefore we multiply ν_1.

Assuming a linear relation between A and P, there are two appropriate forms for it, viz.

$$(A - 8^\circ{\cdot}16) = r \cdot \frac{\sigma_A}{\sigma_P}\left(\frac{P - 311}{50}\right) \text{ and } \left(\frac{P - 311}{50}\right) = r \cdot \frac{\sigma_P}{\sigma_A}(A - 8^\circ{\cdot}16),$$

and those new to correlation methods will notice with surprise that these are not convertible. To save repeating explanations already given in detail elsewhere, we need only remark that to find the most probable value of A when P is given is not the same as to find the most probable value of P when A is given. There is the same *kind* of difference (the analogy must not be taken too seriously) as between finding from the fact that a man is 25 the date of his probable death, and finding from the date of his death the probable date of his being 25: the latter case is clearly complicated by the possibility that he may never have reached 25 at all.

15. In astronomical examples it will frequently happen that one of the variables is better determined than the other, and hence one of the two alternative forms is indicated as preferable. Thus in our case P is well known; most of the periods are known within a day or two, some perhaps to fractions of a day; and since our unit in Table II. was 50 days, the precision is less than one-tenth of the unit. But A may be several degrees wrong, *i.e.* several

whole units. Hence it is natural to determine A from P, and the appropriate equation is

$$(A - 8^{\circ}\cdot16) = 0\cdot56 \times \frac{3\cdot22}{2\cdot00}\left(\frac{P - 311}{50}\right)$$

or

$$A = 2^{\circ}\cdot6 + \cdot018P,$$

P being now measured in *days*, and not in units of 50 days. The most probable values of A corresponding to different periods are thus:

For P = 100^{d}	200^{d}	300^{d}	400^{d}	500^{d}	600^{d}
A = 4°·4	6°·2	8°·0	9°·8	11°·6	13°·4

16. This single example will suffice to show the method of finding r in such cases. There are many other points to be considered, *e.g.* how to deal with cases where the relation is not linear, or where one of the quantities is not measured but only described. But it seems probable that in astronomy the evaluation of r by the above simple process will be the point most often occurring.

17. But before leaving the more general topic of correlation for the particular application of it we have in view, it is a matter of some interest to return now to the simple example given in § 2, and work it out as below.

TABLE IV.

The example given in § 2.

B \ A	5	6	7	8	9	
6	1 (2)	1 (1)	1			3
7		1	1	1		3
8			1	1 (1)	1 (2)	3
	1	2	3	2	1	9

A	Sum.	Factor.	ν_1	ν_2
5	1	−2	−2	4
6	2	−1	−2	2
7	3	0	...	...
8	2	+1	+2	2
9	1	+2	+2	4
Sums	9	...	0	12

B	Sum.	Factor.	ν_1	ν_2
6	3	−1	−3	3
7	3	0	...	...
8	3	+1	+3	3
Sums	9	...	0	6

Product S′(AB)

+3	0
0	+3

= 6

$$\sigma_A = \sqrt{12} = 3\cdot46 \qquad \sigma_B = \sqrt{6} = 2\cdot45$$

$$r = \frac{6}{9 \times \sqrt{12} \times \sqrt{6}} = \cdot08\,.$$

This is a very low value, and on referring to Table III. we see that the probable error is greater than 0·2 : so that the correlation of the quantities A and B may be regarded as unsupported. And yet at first sight there is a fair appearance of relationship. This shows the advantage of having a numerical test.

18. We proceed now to discuss the interpretation of this and similar results. And in the first place it is an obvious suggestion to inquire whether C is also related to P. In a precisely similar manner, which need not be given in detail, we find the correlation for C and P to be

$$r = \cdot 30 \pm \cdot 10 ,$$

and for the probable relationship

$$(C - 27\cdot 2) = \cdot 064(P - 311)$$

or

$$C = 7\cdot 3 + \cdot 064P ,$$

where C and P are measured in days. But the value of r and its probable error show that the probability of a relationship is here more doubtful. This is quite possibly due to the much greater uncertainty of the values of C, which is more difficult to determine than A, for reasons which those who have discussed variable star observations will readily appreciate.

With this formula the values of C would be:

P =	100^d	200^d	300^d	400^d	500^d	600^d
C =	14	20	26	33	39	46

19. Now, having obtained the suggestion of correlation and two formulæ for A and C, I examined in detail some individual cases which had come under notice and which are given below. It will be seen that there is good confirmatory evidence, so far as it goes, of these relationships.

S Cassiopeiæ (Ch. No. 432).

20. In *Mem. R.A.S.*, lv., p. lxix, the Rousdon observations are compared with Chandler's formula in his 3rd Catalogue :

$$610\cdot 5E + 50 \sin (10^\circ E + 50^\circ) .$$

From observations E = 17 to 21 (about) a period of 630·7 days is derived, differing sensibly from 606·2 given by the formula. It is remarked : "To obtain [630·7 days] from the formula we must alter the periodic term in some way, and especially we must increase the coefficient, for with the coefficient 50 the greatest interval between two consecutive maxima is the greatest value of

$$610\cdot 5 + 50[\sin \theta - \sin (\theta + 10^\circ)] ,$$

which is 610·5 + 100 sin 5° or 619·2 days. But we cannot discuss this point without reference to other observations. Chandler used maxima in 1843 and 1863–93, and any new formula must thus

agree with the old one at these periods. Tentatively we may suggest some such modification as

$$2401590 + 615E + 75 \sin (12^\circ E + 90^\circ).$$"

Chandler's revision gives

$$610{\cdot}5E + 37 \sin (15^\circ E + 59^\circ).$$

The value of the periodic term deduced from the period 610 days by the formulæ found is

$$47 \sin (13^\circ{\cdot}6\ E + ?).$$

The coefficient given by the formula is thus in accord with Chandler's first thought, and lies between his revised value and the value suggested very tentatively by the Rousdon observations. The mean of the three different suggested values, 75, 50, and 37, is 54; and the fact that the smallest of these, 37, was used in deducing the formula accounts for part of the difference between the formula value 47 and this mean.

It is perhaps worth noting that if the suggestion of C = 75 (made quite independently of the present investigation) is adopted instead of Chandler's C = 37, the value of r is raised from ·30 to ·43.

The argument 13°·6 given by the formula is between the value suggested by the Rousdon observations and Chandler's revision.

R Ursæ Majoris (Ch. No. 3825).

21. In *Mem. R.A.S.*, lv., p. lxxvi, the Rousdon observations are compared with Chandler's 3rd Catalogue formula

$$302{\cdot}1E + 15 \sin (10^\circ E + 190^\circ).$$

It is shown that the Rousdon observations indicate an error of 150° in the periodic term: so that the coefficient of E should be 8° instead of 10°. Chandler's revision gives

$$302{\cdot}1E + 11 \sin (8^\circ E + 238^\circ).$$

Pogson's observations of this star have since been very carefully discussed by Miss Blagg; and the outcome of this entirely independent discussion (which will shortly be published in *Mem. R.A.S.*, lviii., in the introduction to Pogson's observations) was to indicate a periodic term

$$40 \sin (8^\circ E + ?).$$

The formula gives for 302 days a term

$$27 \sin (8^\circ E + ?),$$

in which the coefficient is midway between Chandler's revision and the Pogson indication; and the argument in good accordance with both.

T Ursæ Majoris (Ch. No. 4511).

22. In *Mem. R.A.S.*, lv., p. lxxviii, the Rousdon observations are compared with Chandler's 3rd Catalogue formula

$$257{\cdot}2\mathrm{E} + 20 \sin (9^\circ\ \mathrm{E} + 90^\circ),$$

with the conclusion that the agreement is good. Chandler's revision gives no change. The correlation formulæ give for a period of 257 days a periodic term 24 sin (7° E + ?).

S Ursæ Majoris (Ch. No. 4557).

23. On p. lxxx the Rousdon observations are compared with Chandler's 3rd Catalogue formula

$$226{\cdot}1\mathrm{E} + 43 \sin (5^\circ{\cdot}76\ \mathrm{E} + 181^\circ{\cdot}5),$$

with the conclusion that "there is a fairly satisfactory accordance, though some correction to the formula would improve it."

Chandler's revision gives

$$226{\cdot}5\mathrm{E} + 35 \sin (5^\circ{\cdot}4\ \mathrm{E} + 194^\circ).$$

The correlation formulæ give for a period of 226·5 days a periodic term 22 sin (6°·5 E + ?), which suggests that Chandler's diminution of the coefficient has not been carried far enough, though in the right direction.

S Cygni (Ch. No. 7220).

24. On p. lxxxix the Rousdon observations are compared with Chandler's formula

$$322{\cdot}8\mathrm{E} + 15 \sin (12^\circ\ \mathrm{E} + 66^\circ).$$

It is remarked that the "periods at (the Rousdon) epoch would agree better if the coefficient of E in the periodic term were smaller, say 9° instead of 12°."

In Chandler's revision the periodic term is replaced by a secular term

$$323\mathrm{E} + 0{\cdot}015\mathrm{E}^2.$$

The correlation formulæ give for a period of 323 days

$$28 \sin (8^\circ\ \mathrm{E} + ?),$$

so that the only suggestion made at the time of discussing the Rousdon observations is in the direction of better accordance with the formulæ.

R Sagittæ (Ch. No. 7257).

25. This star might have been included in Table I., but it was decided to draw the line as regards "long-period variables" at 100 days. The elements given in the revision are

$$70{\cdot}56\mathrm{E} + 6{\cdot}5 \sin (2^\circ{\cdot}25 + 47^\circ).$$

Extrapolating our formulæ, we get C = 12 and A = 3°·9. But if we decide that this star may be regarded as a long-period variable, we should include it in our table, and then the values of C and A would be found closer to those observed, since the star would have great weight. Indeed, the correlation for A and P is raised to $r = 0{\cdot}64 \pm {\cdot}07$, and the formula becomes

$$(A - 8^\circ{\cdot}1) = 1^\circ{\cdot}01\left(\frac{P - 305}{50}\right)$$

or

$$A = 1^\circ{\cdot}9 + {\cdot}020\ P,$$

which gives A = 3°·3 for P = 70.

The value of r for C and P is, however, not much improved, being raised from ·30 to ·32.

R Cassiopeiæ (Ch. No. 8600).

26. On p. xcii of the Rousdon Memoir, the observations are compared with Chandler's 3rd Catalogue formula,

$$429{\cdot}5E + 25 \sin (15^\circ E + 0^\circ),$$

and it is remarked :—

"The corrections to period shown by the different columns of the Rousdon observations are so consistent that it is difficult to believe that the mean result can be so erroneous as the formula would make it. If the formula were altered to

$$431E + 30 \sin (12^\circ E + 9^\circ)$$

[there would be a certain improvement]; but it is of course impossible to alter the formula definitively without discussing other observations."

Chandler's revision, which appeared after the above words were in type, gives

$$431{\cdot}6E + 32 \sin (9^\circ E + 60^\circ).$$

The correlation formulæ give for the period 431 days a periodic term

$$35 \sin (10^\circ{\cdot}7\ E + ?),$$

showing that the improvements suggested by the Rousdon observations were in the right direction, though not sufficient in magnitude; and that Chandler's revision is in good accord with the formula.

S Delphini (Ch. No. 7431).

27. Chandler gives no indication of a periodic term in his "revision," printing the period as 277·5 days. In discussing Baxendell's observations (before the present correlation work had been undertaken at all), Miss Blagg found clear indications of a periodic term at which a preliminary guess of 9 sin ($7\frac{1}{2}^\circ$ E + ?) was made, the 9 being mere guess-work, but the $7\frac{1}{2}^\circ$ being indicated with

fair precision. The correlation formulæ give for a period 277·5 days $A = 7°·6$, which is close to the value guessed. For the coefficient C the formula gives 25 days, which is a good deal larger than the guessed 9; but on *calculating* the coefficient from the separate corrections to epoch the coefficient came out 28·3, which accords well with the formula. Although the calculation is of a rough kind, it seems worth while giving it here to show the kind of accordance that is at present obtainable.

TABLE IV.

Baxendell's *S Delphini*.

E	Observed Correction to Epoch.	Argument ($7\frac{1}{2}°$ E+const.)	Sin (Arg).	Product.
I.	−12d	− 26°	−·44	+ 5·3
II.	− 5	− 18½	−·32	+ 1·6
III.	+ 3	− 11	−·19	− 0·6
IV.	− 5	− 3½	−·09	+ 0·5
V.	+11	+ 4	+·07	+ 0·8
VI.	+20	+ 11½	+·20	+ 0·4
VII.	+21	+ 19	+·33	+ 6·9
VIII.	+ 6	+ 26½	+·45	+ 2·7
IX.	0	+ 34	+·56	0·0
X.	+23	+ 41½	+·66	+15·2
XI.	+38	+ 49	+·76	+28·9
XXIV.	+ 5	+146½	+·55	+ 2·8
XXV.	+ 3	+154	+·44	+ 1·3
XXVI.	+ 1	+161½	+·32	+ 0·3
XXVII.	+ 9	+169	+·19	+ 1·7
XXVIII.	+ 6	+176½	+·09	+ 0·5
XXIX.	−19	+184	−·07	+ 1·3
XXXII.	− 8	+206½	−·45	+ 3·6
XXXIII.	−27	+214	−·56	+15·1
XXXIV.	−13	+221½	−·66	+ 8·6
XXXV.	−29	+229	−·76	+22·0
			Sum	118·9

28. The first column gives the sequence of periods observed by Baxendell. There is a wide gap between XI. and XXIV. with no observations, and it will be seen from columns 3 and 4 that it was unfortunately just in this gap that the corrections to epoch would have been largest, and given us an accurate value. Still, the periods when the correction vanishes are fairly well determined, which is something. They have been assumed to lie midway

between IV. and V., and between XXVIII. and XXIX. This is only a rough assumption, but the material does not warrant refinements. The "observed correction to epoch" given in the second column had been deduced from the observations by Miss Blagg, using a purely numerical process,* before there was any idea of applying them in this way. In the next three columns the product by sin (arg) is deduced, and is almost uniformly positive. Dividing the sum 118·3 by $\Sigma \sin^2(\text{arg})$, which comes out 4·18, we get as above mentioned

$$C = 118{\cdot}3/4{\cdot}18 = 28{\cdot}3.$$

S Serpentis (Ch. No. 5501).

29. For a different reason it is necessary to call attention to the case of S Serpentis, period 369 days. Chandler gives C = 116, A = 4°, while our formulæ indicate C = 30, A = 9°. But it is apparently possible to satisfy the observations in a different way, which assigns to C and A nearly the values of the formula. This investigation is given in a separate note following this paper.

30. What are now wanted are better determinations of C and A for stars with long and short periods. Too much depends on S Cassiopeiæ (610 days) and R Virginis (145 days), in the present state of our knowledge of them. "The dog is wagged by the tail." But it is the special value of an investigation of this kind that attention is directed to special needs which may guide observers in selection. The following stars, for instance, seem worth special attention from the northern observers:—

Short Periods.			Long Periods.		
Name.	No.	P	Name.	No.	P
Z Aquilæ	7260	127d	S Cassiop.	432	610d
W Cygni	7754	131	V Delphini	7458	540
R Vulpec.	7560	137	S Cephei	7779	486
R Virginis	4521	145	W Aquilæ	6900	480
S Aquilæ	7242	147	U Cygni	7299	461
V Capric.	7571	157	Z Sagittarii	6923	452
T Herculis	6512	165	R Leporis	1771	436
R Ceti	845	167	T Draco	6449	426
V Tauri	1717	170	R Cygni	7045	426
R Arietis	782	187	R Hydræ	4826	425

31. Returning now to Table I., in the seventh column is given M − *m*, the interval between maximum and the preceding minimum according to Chandler's revision. For most variables this is less than half the period, so that the quantity

$$\alpha P = 2(M - m) - P$$

* The process can readily be inferred from the description on pp. 66 to 68 of *Mem. R.A.S.*, vol. lv., though this description is adapted to a period, *i.e.* a difference of epochs, instead of to the epoch itself.

shown in the last column is negative. The quantity a was used in *M.N.*, lxvii. p. 350, in connection with the classification of these variables according to their light curves, and it is interesting to see whether the type of light curve is related to the quantities C and A we have been discussing. The value of r for C and aP comes out ·011, showing that there is complete independence. In considering the type of light curve it was early noticed that it seemed to be independent of the period; and it is not surprising therefore that C and A, which seem to depend on the period, should be independent of the type of curve. The relation between A and aP, and those between a alone and C and A, might be actually worked out, but it seems improbable that they will yield any result of interest.

32. Column 6 of Table I., headed "Phase," represents a much more speculative investigation, but one which it seemed just worth undertaking. It seemed quite possible that C and A, or at any rate A, might be *directly proportional to P*. The limits of possible error are such that there is nothing extravagant in this supposition, which would be represented by the equation

$$(A - 8^\circ{\cdot}2) = k\left(\frac{P - 311}{50}\right),$$

where
$$k = \frac{8{\cdot}2 \times 50}{311} = 1{\cdot}32,$$

so that
$$A = {\cdot}0264P.$$

We may compare the resulting values of A with those of § 15.

	P = 100^d	200^d	300^d	400^d	500^d	600^d
A by old equation =	4°·4	6°·2	8°·0	9°·8	11°·6	13°·4
A by new equation =	2·6	5·2	7·8	10·4	13·0	15·6

33. Now what makes this hypothesis of special interest is the fact that the cycle in which the term C sin (A° E + const.) is completed is 360/A periods or 360P/A days, and if P/A is constant we are confronted with the idea of a universal cycle controlling all these stars! The notion seems impossible, and yet there might be a simple explanation of it—the cause might be rooted in us, and not in the stars. It is difficult to see how any feature of the Earth or of the solar system could affect differential comparisons of distant stars, but though difficult it may not be impossible. The period indicated is 360/·0264 days or 37·4 years. If there is a phenomenon of this kind originating with the observer, it might have the same phase for all stars, or at least the phase might depend on the star's position. Hence the phases given by Chandler were reduced to the common epoch 240 5000, and are shown in the sixth column of Table I. They certainly tend to no one value more than another, and a few experiments on their being related to a star's position gave no encouragement.

34. Another point may be mentioned. Whether there is

actually a single periodicity or not, the cycle for the periodic term is about of the same order for different stars. The value of 360P/A in years given by the equation of § 15 is

$P = 100^d$	200^d	300^d	400^d	500^d	600^d
Cycle = 22^y	32^y	37^y	40^y	43^y	46^y

Now if its coefficient also remains of the same order of magnitude, the maximum gradient of the disturbing cause (whatever it may be) may be nearly constant. Suppose it constant for simplicity. Then the displacement of maximum of a variable is due to the addition to the term

$$y_1 = f \cos 2\pi t/P$$

(which represents the ordinary variation near maximum) of a term

$$y_2 = kt,$$

where k may be nearly the same for all stars. The time of maximum will now be when

$$0 = \frac{dy_1}{dt} + \frac{dy_2}{dt} = -\frac{2\pi}{P} f . \sin \frac{2\pi t}{P} + k,$$

or, since t is small,

$$t = \frac{kP^2}{4\pi^2 . f}$$

Now f represents the range of variation, and is not very different for the stars in Table I., being usually about 5 magnitudes. For some stars the range is as low as 3, and for others it is as great as 7; and in a complete investigation the influence of this range must undoubtedly be considered. But for the present we will consider f constant. It follows that C, which is the maximum alteration of epoch, would be proportional to P^2 rather than to P. Hence it is worth inquiring whether the correlation of $\sqrt{C}$ with P is better than that of C with P. On working it out, the result came $r = 0{\cdot}34$, which is indeed greater than the $r = 0{\cdot}30$ previously found, but not much. It would seem that for the present our best work can be done in getting improved values of C and A for individual stars, especially those above given, both by making new observations and by collecting and carefully discussing those already made.

SUMMARY.

§§ 1– 3. Introductory.

§§ 4– 5. Tabulation of Chandler's periodic "inequalities."

§§ 6–17. An example of working out the "correlation" r between two quantities A and P. The result found is $r = 0{\cdot}56 \pm 0{\cdot}08$, which is worth serious attention. The inference is that the argument A° in Chandler's periodic inequalities can be deduced from the period P days by the following probable formula:—

$$A = 8°{\cdot}2 + {\cdot}018P .$$

§ 13 includes a table (III.) for the probable error of r.

§ 18. For C (the coefficient of the periodic inequalities) and P we find $r = 0{\cdot}30 \pm 0{\cdot}10$, which is not so conclusive. The relation would be

$$C = 7{\cdot}3 + {\cdot}064P.$$

§§ 19–29. Examination of 8 stars in detail, for which special information was available. The result is favourable. Remarks on S Serpentis, which seems to be exceptional.

§ 30. List of stars needing special attention.

§ 31. The type of light curve seems to be independent of C and A.

§§ 32–34. Possibility of a single periodic cause affecting all stars considered, but not supported.

Note on the Period of S Serpentis. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The formula given for the maxima of S Serpentis (No. 5501) by Chandler in his Revision of the 3rd Catalogue (*A.J.*, No. 553) is

$$2388724 + 368{\cdot}5\,E + 116 \sin (4^\circ E + 62^\circ).$$

The periodic term in this formula attracted attention by considerable divergence from the value suggested by the preceding paper. The coefficient 116 days is much too large, and the argument 4° E is too small. With a view to seeing whether it was well established, or whether perhaps some other formula would fit the observations, inquiry was made of Professor Müller, of Potsdam, who very kindly sent a complete list of observed maxima, with full references; adding a comparison with Chandler's formula, the remark that it did not fit more recent observations, and a suggestion of his own for improving it, which modified both the coefficient 116 and the argument 4° E in the right direction. But this suggestion does not fit the observation of Lalande, and reasons will be given below why it is probably too early to suggest a completely satisfactory formula, in spite of the fact that Lalande's observation was made in 1794. Hence it seems unnecessary to reproduce here the full details, which will doubtless appear in the great work of reference for variable stars now being prepared by the German Committee of which Dr. Müller is a member.

2. For our present purpose the observations are sufficiently represented by the dates for every fifth maximum shown in the second column of Table I. The numeration in the first column is that of Chandler. In the third column are given the intervals, and we have to decide how to interpret Lalande's observation, which may belong to any one of the epochs -35, -34, or -33. Chandler takes -33; and this gives an average interval of 1876 days for 5 periods, extending over 7×5 periods in all. This was quite a possible interpretation before the modern observations (represented

by epoch 76) were made. Up to epoch 71 the interval was increasing, and might have gone on doing so. But later observations have made this interpretation impossible. The maximum interval is clearly past at epoch 71. We have only a single entry in the table to prove this, but the entry stands for 5 well-determined maxima in which 5 separate observers were concerned: 3 of them observed epoch 74, 2 of them epoch 75, and epochs 76, 77, 78 were all observed. The mean errors of Chandler's formula are −20, −31, −27, −44, −43 days; and it seems clear that the formula no longer holds.

TABLE I.

E.	Max.	Int.	A+365·1 E.	·080×(E−36)².	O−C₁.	−40 sin.(7°·2 E+281°).	O−C₂.
−35	2376442		2376018	403	+21	−24	−3
or −34		7(1772)	383	392			
or −33		or 7(1824)	748	381			
		or 7(1876)					
1	2389210		2389161	98	−49	+38	−11
6	—	3(1835)	—	—	—	—	
11	—		—	—	—	—	
16	2394714		2394638	32	+44	−24	+20
		1813					
21	6527		6463	18	+46	−38	+8
		1807					
26	8334		8289	8	+37	−38	−1
		1798					
31	2400132		2400114	2	+16	−24	−8
		1802					
36	1934		1940	0	−6	0	−6
		1809					
41	3743		3765	2	−24	+24	0
		1818					
46	5561		5591	8	−38	+38	0
		1834					
51	7395		7416	18	−39	+38	−1
		1853					
56	9248		9242	32	−26	+24	−2
		1874					
61	2411122		2411067	50	+5	0	+5
		1860					
66	2982		2893	72	+17	−24	−7
		1873					
71	4855		4718	98	+39	−38	+1
		1854					
76	6709		6544	128	+37	−38	−1
81	(to come)		8369	162		−24	
86	(,,)		2420195	200		0	

On the other hand, if we alter Lalande's observation by one whole period, we get an average interval of 1824 days for 5 periods. This means that a maximum interval occurs between epoch 1 and 16, where the average value is 1835 days. But this maximum is much less than that between epochs 61 and 76, where the average value is 1869 days.

3. The conclusion seems to be that we cannot satisfy the observations by a *single* inequality, however we choose the coefficient and argument, if Lalande's observation is to be accepted. But this is, after all, no new thing. Chandler gives several cases of the kind, and others may be found in the future. For instance, he gives for R Hydræ (No. 4826),

$$2411931{\cdot}0 + 425{\cdot}15\ E - 0{\cdot}36\ E^2 + 15\ \sin\ (7^\circ{\cdot}5\ E + 202^\circ).$$

If we admit an additional term in E^2 (which means that the period is changing steadily), we can fairly satisfy the observations. The approximate value of the coefficient of E^2 is indicated by the fact just quoted, viz. that the average interval near the maximum at epoch 9 is 1835 days (average period 367 days), while that at epoch 69 is 1869 days (average period 374 days). Thus the period changes 7 days in 60 periods, or the coefficient of E^2 is about ·06.

4. But anyone who has had experience of such work knows that such rough approximations are followed by a series of "trials and errors"; and by experiment a better value for the coefficient was found to be ·080, as indicated in the fifth column of Table I. The accompanying value of the period (for E=36) is 365·1 days; and starting with an arbitrary epoch, this is represented in the fourth column.

5. The values of O—C are shown in the sixth column, from which it is seen that an inequality of much shorter period than Chandler's will satisfy them approximately if Lalande's observation be referred, not to E= − 33 or even E= − 34, but to E= - 35. The numerical values assumed are

$$+40\ \sin\ (7^\circ{\cdot}2\ E + 281^\circ),$$

which are in much better agreement with the values assigned by the correlation formulæ of the preceding paper, viz.

$$+30\ \sin\ (9^\circ{\cdot}0\ E + ?),$$

than Chandler's expression,

$$116\ \sin\ (4^\circ\ E + 62^\circ).$$

6. The final residuals are shown in the column O—C_2. We must wait for confirmation of the considerable assumption already made; and to this end the maxima ensuing in the next few years are predicted as—

	J.D.	
79	241 7817	= 1907 Aug. 29
80	241 8186	1908 Sept. 1
81	241 8555	1909 Sept. 5
82	241 8923	1910 Sept. 8
83	241 9291	1911 Sept. 11
84	241 9659	1912 Sept. 13
85	242 0027	1913 Sept. 16
86	242 0395	1914 Sept. 19

7. The complete formula is thus—

$$2388796 + 365{\cdot}1\,E + {\cdot}080\,(E-36)^2 + 40\sin(7^\circ{\cdot}2E + 281^\circ)$$
$$= 2388900 + 359{\cdot}3\,E + {\cdot}080\,E^2 + 40\sin(7^\circ{\cdot}2\,E + 281^\circ);$$

but the term ·080 E^2 is of a provisional character, and probably represents the present value of a term of long period.

8. The main interest of this inquiry is the result that even in such an exceptional case as that of S Serpentis it is found possible to obtain fair accordance with the suggested formula by a simple supposition of a kind already familiar in other cases; while it seems certain that the discrepant formula is not correct. Possibly in other cases of discordance similar explanations may be found.

On the Orbit of β 416. By J. Voûte.

(*Communicated by W. Bowyer.*)

The periods calculated for this interesting binary differ very much. The last, that of Doberck, with a period of forty-six years, was based on the measures up to 1903, and his ephemeris gives, with the recent observations, nearly constant negative differences. In the present instance the measures published up to 1906 have been used. In the first place, all the position-angles and distances are brought up to the epoch 1910·0; then 6 normal-places are formed with position-angles and distances. These normal-places are formed out of the groups marked by square brackets. First a normal-place is formed for every year with weights equal to the number of nights observed by the different observers, and then the normal-place of a group with weights equal to the number of years.

Normal-places.

1877·23	228°·77	1″·79	1898·14	305°·37	1″·85
1889·89	129·19	0·98	1901·77	291·35	2·11
1895·80	322·14	1·34	1905·60	280·08	2·21

The apparent orbit drawn through these normal-places gives, following the graphical method of Dr. Zwiers, the following elements:—

P = 41·47 years	a = 1″·86
T = 1891·45	☊ = 131°·0
n = −8·682	ι = 49°·0
e = 0·552	ϖ = 64°·0

Motion retrograde.

Comparison of computed with observed places.

		θ_{1910}.	θ_{O-C}.	ρ_{O-C}.
		°	°	″
Burnham	1876·52	±239	± 5·8	±0·05
Cincinnati	77·53	222·4	− 5·9	0·00
Russell	77·64	224·2	− 4·0	−0·03
Burnham	88·72	147·4	+ 6·4	+0·69
,,	89·42	134·0	+ 3·7	+0·12
Pollock	89·63	131·8	+ 1·8	−0·08
Sellars	90·60	121·9	+12·8	+0·01
Burnham	91·53	81·2	+ 4·5	−0·05
,,	92·38	24·3	−10·3	+0·05
Sellars	93·55	348·2	− 6·9	−0·06
Barnard	94·56	334·4	− 1·1	+0·25
Sellars	94·57	331·7	− 3·7	−0·11
Comstock	95·60	321·6	− 0·9	−0·45
Sellars	95·62	322·1	− 0·3	+0·07
See	95·74	321·3	+ 0·1	−0·08
Aitken	96·44	314·7	− 1·6	−0·07
Scott	96·46	315·3	− 0·9	+0·09
Sellars	96·59	315·7	+ 0·4	+0·33
Comstock	96·60	314·8	− 0·5	−0·25
See	97·39	306·9	− 3·2	−0·01
Aitken	97·46	309·0	− 0·7	+0·29
Scott	97·47	308·7	− 0·9	+0·09
Lehman	97·52	310·1	+ 0·8	+0·13
Aitken	98·35	303·7	− 1·5	+0·14
,,	99·38	300·1	− 0·5	−0·12
Doolittle	99·42	299·0	− 1·5	−0·18
Aitken	1900·49	295·5	− 0·9	−0·11
Tebbutt	1900·42	298·9	+2·5	−0·07
Lunt	1900·53	298·9	+2·6	+0·14
Doolittle	1900·60	294·1	− 1·9	−0·02
Doberck	1901·76	291·3	−0·4	−0·05
Innes	1902·36	292·0	+2·4	+0·14
Aitken	1902·44	288·8	−0·5	−0·29
Tebbutt	1902·49	289·9	+0·8	+0·38
Aitken	1903·56	286·1	+0·5	−0·13
Doolittle	1903·62	283·3	− 1·1	−0·17
Scott	1903·66	287·1	+1·9	+0·05
Aitken	1904·63	282·8	+1·1	−0·18
,,	1905·63	280·1	+1·2	+0·08
Tebbutt	1905·62	279·3	+0·2	+0·16
Doberck	1905·66	281·5	+2·5	−0·20
Aitken	1906·46	276·9	+0·5	+0·08
Tebbutt	1907·57	274·2	+0·7	−0·12
Scott	1907·56	273·0	−0·5	+0·09

The table shows us that the differences θ_{O-C}, though small, show some period, but a correction of the elements does not

improve them. Better material for correction may be obtained in the next few years.

Ephemeris.

1908·5	270°·3	2″·20	1912·5	256°·4	2″·08
1909·5	267·0	2·18	1913·5	252·6	2·04
1910·5	263·6	2·15	1914·5	248·6	2·00
1911·5	260·1	2·12	1915·5	244·4	1·96

Orbital Elements of Double Stars. By T. J. J. See, Ph.D.

The following orbits have been communicated by Dr. See. The material on which they are based is for the most part to be found in Burnham's *General Catalogue.*

Σ 2.

R.A. $0^h\ 4^m$ Dec. $+79°\cdot 10'$

Discovered by W. Struve in 1828. Always a close pair, and measures are therefore difficult with small or moderate apertures. An orbit was found by Glasenapp using measures to 1893. It was again measured by Biesbroeck in 1904.

P =	166·0 years	☊ =	168°·3
T =	1894·50	ι =	72·3
e =	0·40	λ =	0·0
a =	0″·65		

β 524 = 20 Persei.

R.A. $2^h\ 47^m$ Dec. $+37°\ 55'$

The principal component of the wide pair Σ 318. It was discovered by Burnham in 1378 and is always difficult. It is assumed that Burnham's angle in 1878 should be altered by 180°. All the observations are represented except the distance 0″·22 by Lewis in 1896. It is not probable that the orbit given is more than a good approximation.

P =	36 years	☊	indeterminate
T =	1897·0	ι =	0°·0
e =	0·75	λ =	149·6 = Periastron
a =	0″·16		

Σ 483.

R.A. $3^h 57^m$ Dec. $+39^\circ 14'$

Discovered by W. Struve in 1830. There are in all 20 measures down to 1904, and in 1906 it appeared single to Burnham in the 40-inch. Early observations are important as soon as separation occurs.

P =	135·5 years	☊ =	23°·1
T =	1907·75	ι =	68·0
e =	0·786	λ =	213·4
a =	1″·77		

β 883.

R.A. $4^h 46^m$ Dec. $+10^\circ 54'$

Discovered by Burnham in 1879. A long series of measures is available, and Professor Aitken has communicated those of 1907 and 1908.

P =	16·6 years	☊ =	33°·5
T =	1907·33	ι =	35·7
e =	0·47	λ =	197·3
a =	0″·24		

β 581 AB.

R.A. $8^h 0^m$ Dec. $+12^\circ 35'$

The close pair of a triple system discovered by Burnham in 1878. The distance never exceeds 0″·5, and the object is difficult, but the measures are numerous and good. A few have been rejected. The comes is now passing periastron, and measures, though difficult, will be valuable.

P =	41·2 years	☊ =	133°·7
T =	1909·10	ι =	63·1
e =	0·53	λ =	278·1
a =	0″·61		

AC 5 = 8 Sextantis.

R.A. $9^h 48^m$ Dec. $-7^\circ 38'$

Discovered by Alvan Clark in 1852. The angles given by Dembowski, by the Cincinnati observers in 1878, and by Hall in 1879 have been reversed. There is a systematic run of negative

residuals in angle in the second quadrant, but the available material is not sufficient to justify a modification of the elements.

P =	68·75 years	☊	indeterminate
T =	1882·0	ι =	0°·0
e =	0·60	λ =	240°·6 = Periastron
a =	0″·35		

Σ 3123.

R.A. $12^h\ 1^m$ Dec. +69° 15′

Discovered by W. Struve in 1831. The early measures are few and not very accordant. The early angles have been altered by 180°, as suggested by Lewis. With this proviso an arc of 300° has been described since Struve's measures.

P =	103·3 years	☊ =	56°·9
T =	1860·50	ι =	49·7
e =	0·49	λ =	79·1
a =	0″·32		

ξ Bootis = Σ 1888.

R.A. $14^h\ 47^m$ Dec. +19° 31′

Discovered by Herschel in 1780 and followed regularly. No orbit given as yet represents all the observations. The measures are given in Lewis' *Memoir* and in Burnham's *General Catalogue.*

P =	143·0 years	☊ =	171°·6
T =	1909·36	ι =	45·8
e =	0·546	λ =	34·1
a =	5″·02		

Dembowski 15.

R.A. $16^h\ 41^m$ Dec. +43° 40′

Discovered by Dembowski in 1869. The measures are good, and distributed over an arc of 192°.

P =	96·0 years	☊ =	145°·7
T =	1898·0	ι =	66·05
e =	0·35	λ =	168·3
a =	0″·84		

Σ 2438.

R.A. $18^h\ 56^m$ Dec. $+58°\ 5'$

Discovered by Herschel in 1782, and re-examined the next year. Re-measured by Sir John Herschel in 1830, and afterwards followed by several observers. As periastron was passed about 25 years ago, it will be many years before the present elements can be improved. The eccentricity is higher than that of any other known double star except Σ 2525.

P =	233·0 years	☊	indeterminate
T =	1882·50	ι =	0°·0
e =	0·916	λ =	178°·3 = Periastron
a =	0″·53		

Secchi 2 = Σ 2481 BC.

R.A. $19^h\ 8^m$ Dec. $+38°\ 36'$

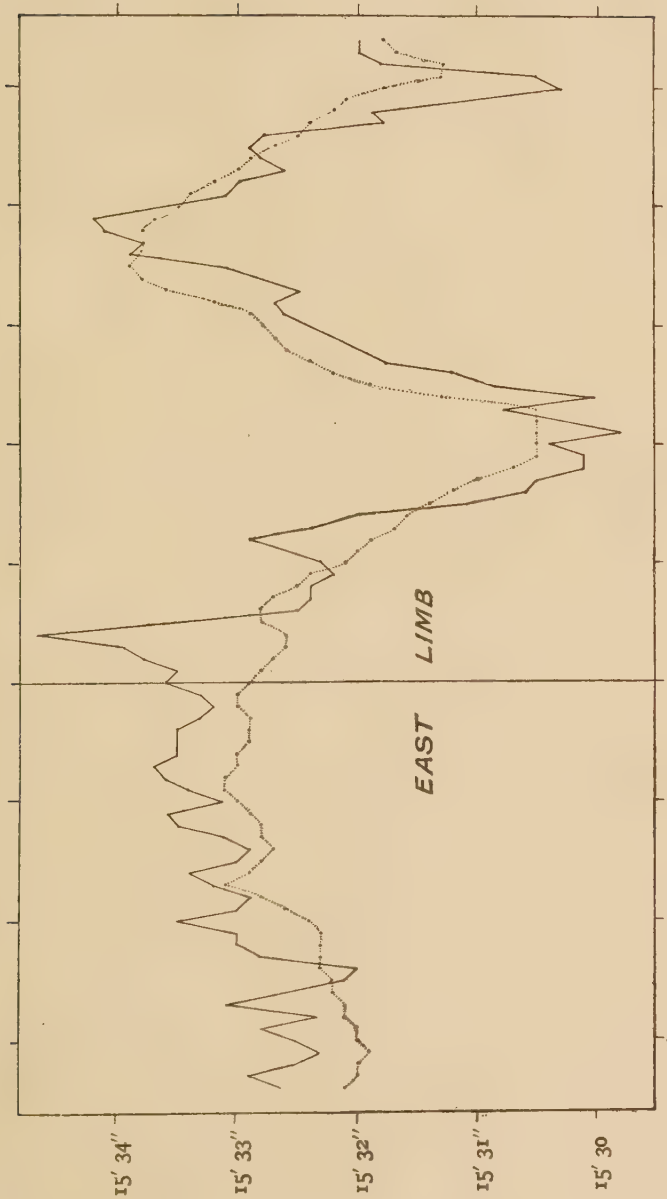
The companion of Σ 2481, found to be a close double by Secchi in 1858. Measured by Otto Struve in 1866, and by Schiaparelli in 1876; there are no measures between 1881 and 1897, since which time it has been followed by Aitken. Aitken's angles before 1901 are altered by 180°.

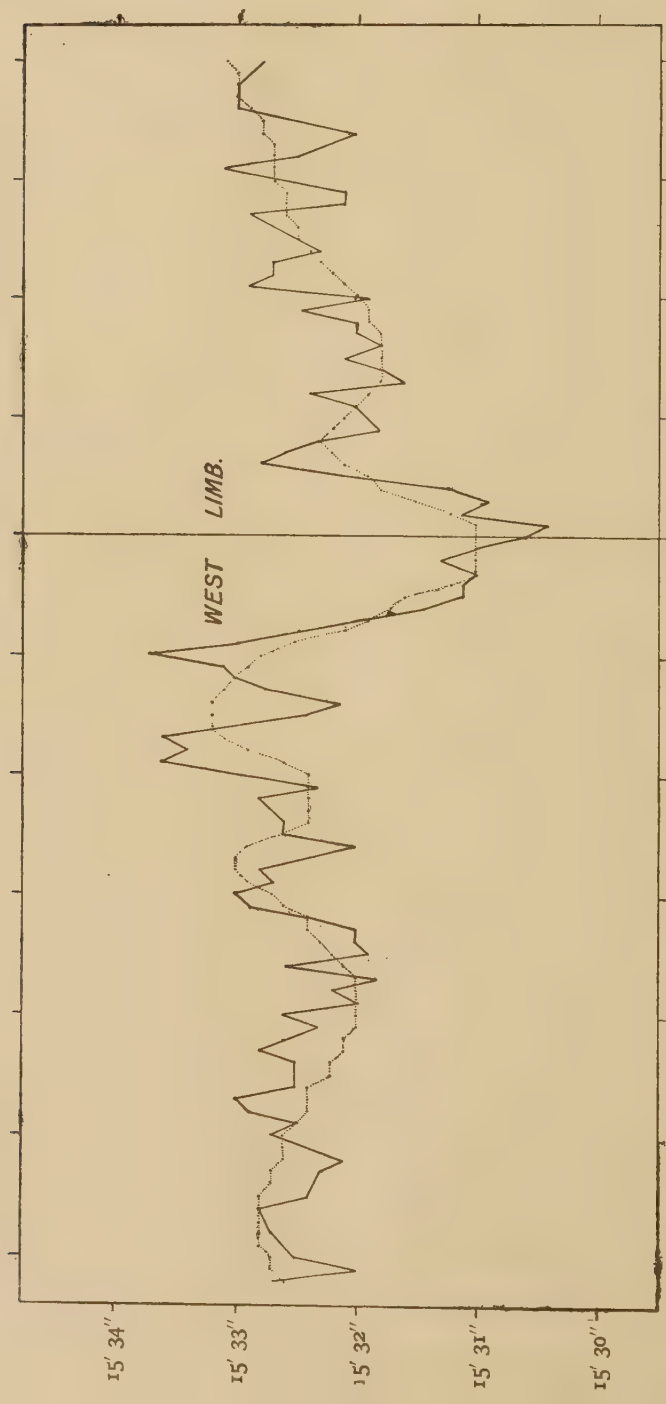
P =	16·0 years	☊ =	109°·7
T =	1902·25	ι =	63°·5
e =	0·68	λ =	218°·3
a =	0″·39		

The Radius of the Moon for libration $-4°·5$. By Walter Heath.

Dr. L. Struve, in his reduction of the occultations observed during the lunar eclipses of 1884 and 1888 (*Dorpat Observatory Publications*, vol. xx.), made a table of the values of the radius of the Moon's limb at different position-angles; a similar table was published by me last year (*Ast. Nach.*, 4206), including more observations, the geocentric libration in longitude being within the limits $-3°·3$ and $-5°·3$.

I have drawn the diagrams below in order to compare the figures in this table with the results obtained by Dr. Hayn from his micrometer measurements at the Leipzig Observatory (*Selenographische Koordinaten*, III Abhandlung, Leipzig, 1907). In the diagrams the figures at the side are for measuring the radius, and those at the bottom denote position-angles; the dotted line shows the values taken from Dr. Hayn's table, and the other line indicates the occultation results. The zero from which the occultation radius is measured is the Moon's centre as determined


15′ 34″
15′ 33″
15′ 32″
15′ 31″
15′ 30
60°
70°
80°
90°
100°
110°
120°
130°
140°
EAST LIMB


15′ 34″
15′ 33″
15′ 32″
15′ 31″
15′ 30″
210°
220°
230°
240°
250°
260°
270°
280°
290°
300°
310°
WEST LIMB.

by Dr. Struve from the occultations of 1884; and in order to compare the curves, I have supposed this point to be the zero for Dr. Hayn's measurements, and that the "mean level" which he adopts as a standard is at 15′ 32″·5 in the diagram.

The occultation curve is compounded of the curves representing the groups of results for 1884 and 1888 and some smaller groups. If the declination of the Moon's centre in respect of any group is corrected, the curve for that group will be turned about the east or west point of the limb; the east and west points for the eclipse of 1884 are at position-angles 114° and 294°, and those for the eclipse of 1888 are at 73° and 253°.

Uplands, Cobham, Surrey:
1908 *June* 7.

The Lunar Bright Rays. By H. G. Tomkins.

It is with some hesitation that I venture to add to the numerous theories which have at one time or another been put forward regarding the lunar bright rays, and my excuse must be the close analogy which exists between them and certain terrestrial phenomena, and the ease with which it can be applied to the Moon.

The present explanation is based, firstly, on the assumption that the configuration of the ray systems may be due to a cause separate from their albedo; secondly, that similar configurations appear to exist on the Earth; and thirdly, that a terrestrial analogy to the white material of the rays is also available.

Taking the albedo first, I am led to think that this can be explained on the supposition that the white material consists of the soluble salts of sodium and other minerals contained in the lunar crust. In the case of the Earth, these salts are brought to the surface by evaporation in a great many parts of the globe. The salts originally exist below the surface, either in the shape of beds of solid salt or salt-impregnated strata; and when they come into contact with the subsoil water, they rise to the surface in solution, and, on the drying up of the water, are left as a white efflorescence. In arid parts such as are found in North India and Persia, the efflorescences are very abundant, and a map of the tracts in the countries named exhibits a distinct tendency towards a radial formation, one of the strips extending about 800 miles towards Bengal from the Salt Range. In Germany also, though, owing to a temperate climate, efflorescences do not occur on a large scale, the brine springs, indicating the presence of the salts, have a distinctly radial configuration. In these cases the white material and the radial formations appear to occur together. This, however, need not always happen, and, as a matter of fact, in many countries the efflorescences do not seem to show any radial tendency, and, on the other hand, radial configurations sometimes appear without efflorescences. This is easily explained when the

origin of the radial configurations on the Earth is considered. They are referred to by several geologists of note, and in particular by Judd in his work on volcanoes, in which he quotes the case of the Lipari Islands, and states that a similar arrangement can be traced in many volcanic regions, especially those in which a great central volcano has existed. They are due to intrusion of liquid matter from below, which, by pushing up a centre of volcanic activity, causes the formation of radial fissures, and upheavals of strata round it. They are not always visible, however, on the surface.

Consequently, if such a system of upheavals occurred in the neighbourhood of saline deposits, the saline strata would be pushed upwards along with other strata, and this would bring them within the reach of evaporation at those places, and result in efflorescences on the surface which would follow the direction of the upheavals, and a radial formation would thus occur. Judd also mentions the presence of saline springs in some cases.

On the Moon, all the conditions of such a state of affairs exist. Careful observation very quickly shows that the ray systems differ greatly in configuration from one another—in fact, their prominent central formations and their albedo are about the only real points of resemblance, and this leads to the conclusion that their albedo may differ in origin from their configuration. As regards the latter, the central formation, the radial configuration of the streamers, the complex forms round Copernicus, and the differences of the systems between themselves, all point to an origin similar to that of volcanic centres on the Earth. Experiments made by Nasmyth and Carpenter and many others also testify to the probability of radial formations accompanying central upheavals or pressure.

Sir George Darwin has demonstrated the probability of the Moon having once been a part of the Earth, and it is reasonable, therefore, to assume that it has a similar constitution; in which case the existence of the salts of sodium and other minerals would be expected on the Moon—more especially sodium, which spectroscopic evidence shows to be present on the Sun and other heavenly bodies. Water does not exist in its free state on the lunar surface, but it is admitted that there is no reason why water should not have existed in the soil; and with the lunar climate, there would therefore have been cause for strong evaporation at some time, even if it does not still continue.

Consequently, any soluble salts in the lunar crust would come to the surface in the shape of efflorescences, and, owing to the absence of clouds or rain which might temporarily operate to reverse the process of evaporation, as on the Earth, the salts would rise more or less continuously, would remain permanently on the surface, and would follow the configurations of the ground, as already explained. If on the Earth water and rain were to disappear from the surface, a similar state of things would inevitably ensue; and in addition to the countries now affected, the areas occupied by the ocean would also have to be taken into account, with the enormous quantities of salts at present in solution.

The only objection of importance to the above explanation is the observed late appearance of the lunar rays at sunrise, and their early disappearance at sunset, which has been regarded by some as indicating physical change of the material of the rays. Such change would not, of course, occur with the salts. The facts, however, that the rays can easily be seen during a total eclipse of the Moon as well as on the dark part of the young Moon, that there are some exceptions to the phenomena, and that, if the site of a ray is watched, sunrise can be seen to extend to it exactly in the same manner as to other formations, are evidence against any physical change, and indicate that their invisibility is probably merely due to the fact that there is little or nothing except their albedo to notify their presence to the eye, and that they are there all the time, and only *seem* to appear when the illumination is sufficient to produce a contrast between the white material of which they are composed and the surrounding country.

I may also add that experiments made by me on this assumption entirely agree with Professor W. H. Pickering's observations that the appearance and disappearance of the rays take place when the solar altitude is from 5 to 10 degrees.

For purposes of the explanation of the rays, therefore, these phenomena can be put down to variations of illumination, and neglected.

Canterbury:
1908 *June* 3.

Observations of Daniel's Comet (d 1907) at the Radcliffe Observatory, Oxford.

(*Communicated by the Radcliffe Observer.*)

The following observations were made with the 10-inch Barclay Equatorial, using the Grubb wire-micrometer with power 200.

Observer—Mr. W. H. Robinson.

Date.	G.M.T.	Local Sidereal Time.	Comet *minus* Star, (Corrected for Refraction only). R.A.	Comet *minus* Star, (Corrected for Refraction only). N.P.D.	No. of Comps.	Apparent R.A. of Comet.	Corrections for Parallax in R.A. *p*.	Log. ($p \times \Delta$).	Apparent N.P.D. of Comet.	Corrections for Parallax in N.P.D. *q*.	Log. ($q \times \Delta$).	Ref.
1907.	h m s	h m s	m s	′ ″		h m s	s		° ′ ″	″		
July 30	14 40 54	23 6 9	−0 23·64	...	12	4 15 13·88	−0·48	9·5658	...	...	...	(*a*)
30	14 45 31	23 10 48	...	−1 32·2	15	...	...	...	74 15 45·4	−8·2	0·7964	(*b*)
Aug. 11	15 6 44	0 19 22	+1 46·77	+1 10·6	4	6 8 1·20	−0·48	9·5798	72 37 17·1	−8·1	0·8110	(*c*)
13	15 49 58	1 10 37	−2 34·35	...	2	6 26 19·65	−0·46	9·5721	...	...	...	(*d*)
13	15 55 2	1 15 42	...	+0 3·2	2	...	...	...	72 42 12·2	−7·6	0·7931	(*e*)
13	16 3 55	1 24 36	−4 2·97	−1 17·3	1	6 26 24·67	−0·45	9·5661	72 42 12·7	−7·5	0·7889	(*f*)
18	15 38 57	1 19 16	+0 34·79	−0 25·1	8	7 9 6·57	−0·43	9·5785	73 15 14·9	−7·4	0·8134	(*g*)
20	15 57 13	1 45 29	+1 15·29	...	5	7 25 24·06	−0·41	9·5762	...	...	...	(*h*)
20	15 58 35	1 46 51	...	−3 8·8	3	...	...	...	73 35 32·5	−7·1	0·8093	(*i*)
26	15 48 44	2 0 37	−5 50·15	−3 3·5	3	8 10 54·23	−0·37	9·5750	74 53 6·1	−6·6	0·8255	(*j*)
Sept. 8	16 16 24	3 19 38	+5 56·09	−3 33·4	1	9 37 25·18	−0·29	9·5672	78 40 40·6	−5·3	0·8335	(*k*)
8	16 41 24	3 44 42	+3 45·77	−7 4·4	4	9 37 32·08	−0·29	9·5683	78 40 59·2	−5·2	0·8258	(*l*)
11	16 41 14	3 56 21	−7 38·65	+7 44·9	2	9 55 19·48	−0·27	9·5671	79 40 32·0	−5·0	0·8293	(*m*)
11	16 47 46	4 2 54	−3 16·37	+1 35·7	1	9 55 20·33	−0·27	9·5671	79 40 39·8	−5·0	0·8276	(*n*)

Observer's Remarks.

(*a*) (*b*) The nucleus or condensation of comet (magnitude 8, and diameter 2″ or 3″) shows up on the preceding side of Coma, the radius of which is about 1′. For a long time after these observations were taken the comet continued to be easily seen in strong twilight.

(*c*) Comet conspicuous to the naked eye, magnitude 2. Tail precedes, 2° or 3° in length. In the 10-inch, the condensation, though large and diffused, is not brighter than magnitude 6·5.

(*d*, *e*, *f*) To the naked eye the comet is nearly as bright as γ Geminorum, magnitude 2. The tail can be traced to a distance approximately equal to that of Pollux from Castor. In the 10-inch the condensation appears large, diffused, and somewhat triangular in form, with a brightness much below that of the second comparison-star, magnitude 7·8 (*A.G.*), but superior to that of the first, magnitude 9·0.

(*g*) Condensation diffused and large, not brighter than the comparison-star, magnitude 8. To the naked eye, tail appeared 7° or 8° long, and nearly reaching to γ Geminorum.

(*h*, *i*) Images very diffused this morning. A nucleus occasionally shows up, slightly south of the observed centre of condensation, which spreads over many seconds of arc. The tail of the comet extends to about midway between the nucleus and γ Geminorum. Altitude small; twilight coming on.

(*j*) Observed through haze in intervals of cloud. The comet's tail is distinctly visible to the naked eye, notwithstanding low altitude, haze, moonlight, and twilight.

(*k*) Altitude of comet only 6°.

(*k*, *l*) Condensation large and diffused.

(*m*, *n*) Comet very low. The diffused condensation is slightly brighter than the second comparison-star, 7·2 magnitude, but much fainter than the first, 6·0 magnitude. The comet was just visible in the 10-inch at $17^h\ 10^m$ G.M.T.

On September 19, the comet was seen for a short interval at a low altitude; magnitude in the 10-inch, 7·5. Observations were prevented by the rapidly increasing twilight, and by the remoteness of the only available comparison-star, which preceded the comet $8^m\ 30^s$ in R.A.

[Table

Assumed Places of the Comparison-Stars.

Ref.	Mean R.A. 1907·0.	Reduction to Appar. R.A.	Mean N.P.D. 1907·0.	Reduction to Appar. N.P.D.	Authority.
	h m s	s	° ′ ″	″	
(*a*) (*b*)	4 15 37·21	+0·31	74 17 16·8	+0·8	Küstner, *Bonn Veröff.*, No. 4, p. 97.
(*c*)	6 6 14·32	+0·11	72 36 4·7	+1·8	Berlin, A (*A.G.*) 1973
(*d*, *e*)	6 28 53·94	+0·06	72 42 7·1	+1·8	Berlin, A (*A.G.*) 2231
(*f*)	6 30 27·59	+0·05	72 43 28·2	+1·8	Berlin, A (*A.G.*) 2252
(*g*)	7 8 31·76	+0·02	73 15 38·2	+1·8	Berlin, A (*A.G.*) 2702
(*h*) (*i*)	7 24 8·76	+0·01	73 38 39·6	+1·7	Berlin, A (*A.G.*) 2862
(*j*)	8 16 44·43	−0·05	74 56 8·1	+1·6	Berlin, A (*A.G.*) 3303
(*k*)	9 31 29·11	−0·02	78 44 12·8	+1·2	Leipzig, I (*A.G.*) 3809
(*l*)	9 33 46·34	−0·03	78 48 2·5	+1·1	Leipzig, I (*A.G.*) 3820
(*m*)	10 2 58·18	−0·05	79 32 46·1	+1·0	Leipzig, I (*A.G.*) 3952 (P. M.'s −0ˢ·0082 and +0″·038.
(*n*)	9 58 36·74	−0·04	79 39 3·1	+1·0	Leipzig, I (*A.G.*) 3933

In the computation of the parallaxes the adopted value of the Sun's mean horizontal parallax is 8″·80, and the geocentric distances Δ are taken from the *Astronomische Nachrichten*, Nos. 4191, 4194, and 4196.

Radcliffe Observatory, Oxford:
1908 *May* 30.

The Orbit of Jupiter's Eighth Satellite. By P. H. Cowell and A. D. Crommelin.

In this paper the time is measured in units of 16 days from 1908 January 12·0 Paris mean time. The astronomical unit of length is used.

Paris mean time has been used because the Connaissance des Temps gives both heliocentric and geocentric longitudes and latitudes of Jupiter referred to the same equinox, the true equinox of date.

With 16 days as the unit of time,

$$\log \text{mass of Sun} = 8{\cdot}87\,943$$
$$\log \text{mass of Jupiter} = 5{\cdot}85\,934$$

It will be seen in the later parts of this paper that the Sun's disturbing force varies during the early part of 1908 between 10 and 6 per cent. of the attractive force of Jupiter, and is therefore by no means negligible. Moreover, if the attempt were made to express the co-ordinates of the satellite with the help of Delaunay's algebraical lunar theory, it would be found that Delaunay's quantities e, e' and m take such large arithmetical values that Delaunay's series are insufficient for the purpose. Hence it is

best to calculate the orbit by quadratures, discarding the idea of an ellipse.

Let the jovicentric co-ordinates at time $n+t$ referred to three rectangular directions be

$$x_n + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + a_6 t^6 +$$
$$y_n + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 + b_6 t^6 +$$
$$z_n + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5 + c_6 t^6 +$$

then we have $x_{n+1} = 2x_n - x_{n-1} + 2a_2 + 2a_4 + 2a_6$.

Again, given the co-ordinates at any time, the accelerations can be calculated in accordance with Newton's law of gravitation. If X denote the x acceleration at time $n+t$,

$$X = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3 + 30a_6 t^4.$$

Hence $X_n = 2a_2$

and $\frac{1}{12}(X_{n+1} - 2X_n + X_{n-1}) = 2a_4 + 5a_6 + \ldots$

The left-hand side of this last equation may be described in words as one-twelfth of the second difference of X_n, and it will be denoted for brevity by P_n.

Hence we have $x_{n+1} = x_n + (x_n - x_{n-1}) + X_n + P_n$

with one error of $3a_6$, which we shall assume to be negligible.

Now let us assume that x_{n-2}, x_{n-1}, x_n have been calculated, and X_{n-2}, X_{n-1}, X_n, and therefore also P_{n-2}, P_{n-1}. We cannot at present calculate P_n, for this requires a knowledge of X_{n+1}, and therefore of $x_{n+1}, y_{n+1}, z_{n+1}$; but we may guess at the value of P_n with the help of the values of P_{n-2}, P_{n-1} (or for the first few values of n, where P_{n-2} has not been calculated, by trial and error); we then obtain a provisional value of x_{n+1} (and similarly y_{n+1}, z_{n+1}); hence we calculate X_{n+1}, and so obtain a revised value of P_n; the correction ΔP_n is then added to x_{n+1}, and in practice it is found that $X_{n+1}, Y_{n+1}, Z_{n+1}$ are unaffected.

We give a numerical application of the above method, taken from our calculations of a provisional orbit. In the illustration it is assumed that the orbit has already been calculated as far as $t=5$ (April 1); it is required to extend it to $t=6$ (April 17). For the sake of clearness, quantities already calculated are written without brackets, quantities guessed at are written in round brackets, and fresh quantities calculated accurately are enclosed in square brackets.

$x_5 = +\cdot0511\ 598$	$y_5 = -\cdot1691\ 203$	$z_5 = -\cdot0173\ 081$
$x_5 - x_4 = -\cdot0160\ 908$	$y_5 - y_4 = -\cdot0004\ 222$	$z_5 - z_4 = +\cdot0095\ 069$
$X_5 = -\cdot0005\ 159$	$Y_5 = +\cdot0020\ 967$	$z_5 = +\cdot0002\ 300$
$(P_5) = +\cdot0000\ 027$	$(Q_5) = +\cdot0000\ 015$	$(R_5) = -\cdot0000\ 020$

Hence by addition,

$$(x_6) = +\cdot 0345\ 558, \quad (y_6) = -\cdot 1674\ 443, \quad (z_6) = -\cdot 0075\ 732.$$

Hence we calculate X_6, Y_6, Z_6, and hence

$$[P_5] = +\cdot 0000\ 029, \quad [Q_5] = +\cdot 000\ 014, \quad [R_5] = -\cdot 0000\ 020.$$

Hence applying the small differences between the true and provisional values of P_5, Q_5, R_5,

$$[x_6] = +\cdot 0345\ 560, \quad [y_6] = -\cdot 1674\ 444, \quad [z_6] = -\cdot 0075\ 732.$$

The foregoing sufficiently explains how to calculate the motion from assumed initial positions; we now deal with the problem how to find an approximate orbit from the observations.

Retaining the previous notation, and interpolating a little among the observations, we shall assume as known the geocentric angular positions at three equidistant times $n-\tau$, n, and $n+\tau$.

Reasoning exactly as before we have

$$x_{n+\tau} - 2x_n + x_{n-\tau} = X_n\tau^2 \text{ with an error of } 2a_4\tau^4.$$

If we take the x direction at right angles to the geocentric directions of the satellite at times $n-\tau$ and $n+\tau$, then our ignorance of the geocentric distances at these times produces no uncertainty in the numerical values of $x_{n+\tau}$ and $x_{n-\tau}$; we therefore have one unknown quantity only occurring, viz. the geocentric distance at time n; and the above equation, by trial and error, will determine this quantity.

The corresponding equations for y and z then determine the geocentric distances on the first and third occasions.

We may then revise our solution so as to correct for the residuals $2a_4\tau^4$, etc.

In practice it is not necessary to take the x axis at right angles to the first and third geocentric distances; we shall then have to deal with three simultaneous equations for three unknown geocentric distances. The idea of resolution perpendicular to the outer geocentric distances is perhaps useful for forming a distinct mental conception of the process.

We shall not go into numerical details in connection with the method just sketched. It may be anticipated, however, that the solution is a dual one, or that two orbits which may be distinguished as direct and retrograde can be found to satisfy three observations. It unexpectedly turned out that the jovicentric distance on February 28 was less on the supposition of a direct orbit than for a retrograde orbit; but a very large eccentricity resulted from the former hypothesis. For this reason combined with the reasons stated in *M.N.*, lxviii. p. 457, the retrograde solution was chosen as the one to be worked up. It will be seen that the corresponding orbit closely follows the observations. Now the dual solution implies that there are two distinct ways of making the coefficients of the squares of the time agree with observation. Of these two solutions the retrograde solution makes the coefficients of the third and fourth

powers of the time also agree with observation. Of course it is not inconceivable that the direct solution might also give cube and fourth terms agreeing with observation, but it is *a priori* distinctly improbable. We have not, therefore, examined the direct solution (or the question of a possible planetary solution) with any minuteness. However closely a direct solution was found to fit the observations, it would be impossible at the present moment to reject the possibility that the retrograde solution was the right one.

The following table contains full details of our provisional orbit. The axes are drawn as follows :—x towards the first point of Aries for 1908·0, y towards longitude 90°, z towards the pole of the ecliptic

$x\ y\ z$ are the jovicentric co-ordinates of the satellite,
$x'\ y'\ z'$ „ „ „ Sun
$X_1\ Y_1\ Z_1$ are the accelerations of Jupiter on the satellite
$X_2\ Y_2\ Z_2$ „ „ the Sun „ „
$X_3\ Y_3\ Z_3$ „ „ the Sun on Jupiter reversed
X Y Z are the sums of the three preceding quantities.
P Q R are the twelfth parts of the second differences of X Y Z.

The values of P, Q, R for April 17 are enclosed in brackets to indicate that their values have been inferred, and not calculated : the values of x, y, z for May 3 are enclosed in brackets to indicate that they depend upon the preceding values of P, Q, R.

Table exhibiting Details of Calculation of provisional Orbit of J. VIII.

Jan. 28.	Feb. 13.	Feb. 29.	Mar. 16.	Apr. 1.	Apr. 17.	May 3.
+·1113290	+·0974232	+·0827044	+·0672506	+·0511598	+·0345560	(+·0175977)
−·1570880	−·1624748	−·1664110	−·1686981	−·1691203	−·1674444	(−·1634199)
−·0531070	−·0447827	−·0359985	−·0268150	−·0173081	−·0075732	(+·0022697)
+3·2999	+3·3949	+3·4882	+3·5797	+3·6694	+3·7572	+3·8432
−4·1496	−4·0791	−4·0065	−3·9318	−3·8551	−3·7764	−3·6960
−0·0587	−0·0611	−0·0635	−0·0658	−0·0681	+0·0704	−0·0727
−·0010107	−·0009553	−·0008821	−·0007863	−·0006613	−·0004987	
+·0014262	+·0015931	+·0017749	+·0019724	+·0021861	+·0024163	
+·0004821	+·0004391	+·0003839	+·0003135	+·0002237	+·0001093	
+·0018109	+·0018614	+·0019080	+·0019507	+·0019889	+·0020229	
−·0022675	−·0022109	−·0021515	−·020899	−·0020262	−·0019611	
−·0000032	−·0000092	−·0000154	−·0000217	−·0000279	−·0000341	
−·0016771	−·0017204	−·0017625	−·0018036	−·0018435	−·0018823	
+·0021090	+·0020670	+·0020243	+·0019809	−·0019368	+·0018920	
+·0000298	+·0000310	+·0000321	+·0000332	+·0000342	+·0000353	
−·0008769	−·0008143	−·0007366	−·0006392	−·0005159	−·0003581	
+·0012677	+·0014492	+·0016477	+·0018634	+·0020967	+·0023472	
+·0005087	+·0004609	+·0004006	+·0003250	+·0002300	+·0001105	
...	+·0000013	+·0000016	+·0000022	+·0000029	(+·0000036)	
...	+·0000014	+·0000014	+·0000015	+·0000014	(+·0000014)	
...	−·0000010	−·0000013	−·0000016	−·0000020	(−·0000025)	

From the co-ordinates x, y, z of the preceding table, the tabular differences of geocentric right ascension and declination between the satellite and Jupiter were obtained. We exhibit also the interpolation for every fourth day.

Table for Paris Mean Noon, giving excess of Satellite's tabular R.A. and Decl. over that of Jupiter.

Date. 1908.	R.A. s	Decl. ″	Date.	R.A. s	Decl. ″
Jan. 28	−7·29	−2588·7	Mar. 20	+112·71	−1419·6
Feb. 1	+1·19	2511·9	24	122·83	1332·4
5	9·72	2430·4	28	132·99	1247·7
9	18·34	2345·0	Apr. 1	143·15	1165·7
13	27·07	2256·4	5	153·27	1086·7
17	35·96	2165·3	9	163·32	1010·7
21	45·01	2072·2	13	173·25	937·8
25	54·22	1977·8	17	183·02	868·0
29	63·60	1883·0	21	192·60	801·4
Mar. 4	73·15	1788 3	25	201·95	737·8
8	82·85	1694·0	29	211·03	677·3
12	92·69	1602·3	May 3	+219·81	−619·7
16	+102·65	−1509·3			

The next table gives the dates of the observations reduced to Paris time with the light times subtracted; the place of observation, and the observed geocentric distances in R.A. and declination of the satellite from Jupiter; also the excess of the corresponding calculated co-ordinates over the observed.

Comparison of Provisional Orbit with Observation.

Date.	Hour.	Place.	Observed R.A. J. VIII—$\mathcal{Y}$.	Calculated minus Observed.	Observed Decl. J. VIII—$\mathcal{Y}$.	Calculated minus Observed.
1908.	h		s	s	″	″
Jan. 27	12·2	Greenwich	− 8·29	−0·04	−2597·3	−0·6
Feb. 1	11·4	,,	+ 2·20	0·00	2501·1	−1·3
3	10·0	,,	6·37	−0·04	2464·2	+1·0
22	10·5	,,	48·38	−0·07	2038·1	−0·3
23	8·1	,,	50·31	+0·07	2015·7	−1·4
24	12·0	,,	53·15	−0·09	1989·2	−0·4
27	10·5	,,	59·94	−0·02	1919·7	−0·4
28	11·0	,,	62·34	−0·02	1895·2	−0·7
Mar. 3	9·3	Heidelberg	71·72	−0·03	1804·6	+1·8
8	19·9	Lick	84·72	+0·16	1674·4	−0·4

Comparison of Provisional Orbit with Observation—continued.

Date.	Hour.	Place.	Observed R.A. J. VIII—$\mathcal{U}$.	Calculated minus Observed.	Observed Decl. J. VIII—$\mathcal{U}$.	Calculated minus Observed.
1908.	h		s	s	"	"
Mar. 23	7·1	Heidelberg	121·12	−0·07	1347·1	−0·5
27	8·4	Greenwich	131·35	−0·01	1259·9	−1·4
31	8·8	,,	141·55	−0·01	1178·1	−0·4
Apr. 1	16·3	Lick	145·00	−0·13	1153·7	+1·6
3	9·3	Greenwich	149·33	−0·14	1120·2	+2·0
24	9·2	,,	201·03	−0·52	1745·7	−1·7
29	16·3	Lick	+213·08	−0·55	− 669·9	+2·6

It will be noticed that the accordance is exceedingly close, considering that this is a first approximation to the orbit, and that nothing whatever has been done in the way of modifying the orbit to produce a better accordance.

It remains to modify the orbit, and then to carry it forward to the end of the year, to assist in picking up the satellite at the next opposition of Jupiter.

For general interest, from the first three lines of the first table of this paper the following jovicentric longitudes, latitudes, and distances are computed, also L the longitude in the orbit reckoned from the ascending node on the ecliptic.

1908.	Long.	Lat.	L.	Distance.
Jan. 28	305°·32	−15°·42	328°·42	·1997
Feb. 13	300·95	−13·31	333·18	·1946
29	296·43	−10·96	338·17	·1893
Mar. 16	291·74	− 8·40	343·45	·1836
Apr. 1	287·83	− 5·59	349·09	·1775
17	281·66	− 2·54	355·09	·1711
May 3	276·15	+ 0·79	1·53	·1644

The instantaneous orbit is inclined 31°·14 to the ecliptic, and 29°·84 to Jupiter's orbit, and the instantaneous eccentricity has a value about $\frac{1}{3}$; the mean distance is about 0·1702, and the period about two years and two months.

The longitude of ascending node is 277°·46.

Observations of Jupiter's Sixth, Seventh, and Eighth Satellites from photographs taken with the 30-inch Reflector at the Royal Observatory, Greenwich, in 1907–8.

(*Communicated by the Astronomer Royal.*)

The sixth, seventh, and eighth satellites of Jupiter have been under observation during the recent opposition of Jupiter with the 30-inch reflector, photographs having been obtained whenever possible. In all 38 photographs of Satellite VI, 21 photographs of Satellite VII, and 13 photographs of Satellite VIII have been secured between 1907 November 13 and 1908 April 24.

The positions of the satellites have been measured on the photographs taken with the reflector with reference to faint comparison stars (of eleventh or twelfth magnitude), symmetrically distributed about the satellites. The positions of these faint comparison stars were then measured relatively to the reference stars (of eighth to ninth magnitude) on photographs (with two exposures of 20^m or 30^m) taken with the Astrographic 13-inch refractor. The field, sensibly free from distortion, being much larger with this telescope than with the reflector, from 24 to 39 reference stars were available on each plate. As Jupiter moved slowly, it was possible to make one reference plate serve for several photographs, which were each referred to it. The constants were determined in the usual manner, all the stars on the plate given in the Astronomische Gesellschaft catalogues being used for this purpose. Right ascensions and declinations of the satellites were then determined, and compared with the corrected tabular places of Jupiter after applying the corrections

$$\text{in R.A.} + 0^s\cdot110, \quad \text{and} \quad \text{Dec.} + 0''\cdot72$$

as determined below.

The G.M.T. is the arithmetical mean of the times of beginning and end of exposure.

SATELLITE VI.

Date and G.M.T.				Exposure.	Apparent R.A.			Apparent Dec.			Sat. VI—Jupiter. R.A.		Sat. VI—Jupiter. Dec.	
1907.	d	h	m	min.	h	m	s	°	′	″	m	s	′	″
Nov.	13	13	47·8	60	8	59	54·16	+17	27	6·2	−3	10·90	+ 3	40·9
,,	15	14	57·9	70	9	0	11·88	17	24	54·3	3	18·89	+ 2	52·2
Dec.	5	14	15·7	40	9	0	38·63	17	15	14·9	4	14·48	− 5	24·2
,,	5	16	1·7	140	9	0	38·31	17	15	15·7	4	14·52	5	25·6
,,	10	15	7·3	66	9	0	2·87	17	16	17·9	4	21·25	7	34·7
,,	13	13	14·8	13	8	59	34·29	17	17	34·1	4	23·89	8	49·6
,,	14	14	48·9	120	8	59	22·64	17	18	7·9	4	24·44	9	17·8

SATELLITE VI—*continued*.

Date and G.M.T.				Exposure.	Apparent R.A.	Apparent Dec.	Sat. VI—Jupiter. R.A.	Sat. VI—Jupiter. Dec.
1908.	d	h	m	min.	h m s	° ′ ″	m s	′ ″
Jan.	3	13	11·6	120	8 53 41·16	17 39 26·7	4 7·05	17 29·6
,,	9	12	41·9	16	8 51 20·86	17 49 11·6	3 51·03	19 36·9
,,	11	13	10·6	90	8 50 30·68	17 52 44·3	3 44·50	20 17·8
,,	11	14	12·8	15	8 59 29·61	17 52 48·9	3 44·35	20 18·6
,,	27	12	41·4	65	8 43 24·62	18 23 58·9	2 35·64	24 20·6
Feb.	1	11	52·1	112	8 41 9·34	18 34 15·4	2 9·24	25 2·8
,,	1	13	13·6	15	8 41 7·84	18 34 22·5	2 8·92	25 3·1
,,	3	10	27·3	80	8 40 17·18	18 38 17·2	1 58·46	25 13·3
,,	3	11	28·4	25	8 40 16·09	18 38 21·9	1 58·18	25 14·2
,,	6	11	35·4	30	8 38 56·58	18 44 32·9	1 41·12	25 26·0
,,	12	8	16·8	10	8 36 28·09	18 56 20·2	1 6·66	25 29·5
,,	12	9	25·1	10	8 36 26·91	18 56 28·1	1 6·39	25 27·2
,,	22	10	56·6	100	8 32 40·70	19 15 22·3	0 6·44	24 26·3
,,	23	8	33·8	42	8 32 22·58	19 16 57·2	−0 1·20	24 16·8
,,	24	12	24·9	80	8 31 59·88	19 18 58·7	+0 5·55	24 2·5
,,	27	10	59·4	70	8 31 5·65	19 23 53·5	0 22·59	23 24·9
,,	28	10	2·9	14	8 30 48·92	19 25 27·2	0 28·04	23 10·7
,,	28	11	28·6	80	8 30 47·89	19 25 33·0	0 28·36	23 9·6
,,	29	9	39·8	50	8 30 32·32	19 27 0·4	0 33·57	22 56·3
Mar.	9	8	40·4	15	8 28 28·12	19 39 48·8	1 21·33	20 8·2
,,	9	10	13·8	30	8 28 27·40	19 39 54·0	1 21·64	20 6·6
,,	9	10	55·6	9	8 28 27·03	19 39 56·0	1 22·74	20 6·1
,,	19	8	41·9	13	8 27 10·46	19 50 43·5	2 6·70	16 3·2
,,	21	9	48·7	120	8 27 2·82	19 52 29·2	2 14·75	15 6·9
,,	23	8	59·0	120	8 26 58·20	19 54 1·4	2 22·12	14 11·3
,,	27	8	54·3	43	8 26 56·53	19 56 38·1	2 35·44	12 16·0
,,	28	9	11·1	180	8 26 57·85	19 57 12·5	2 38·54	11 45·0
,,	31	9	16·4	165	8 27 5·78	19 58 36·7	2 46·98	10 14·5
Apr.	3	9	51·7	90	8 27 19·81	19 59 38·9	2 54·31	− 8 40·7
,,	20	9	21·0	18	8 30 24·95	19 58 4·9	3 12·34	+ 0 11·6
,,	24	9	43·5	100	8 31 32·95	+19 55 47·5	+3 10·54	+ 2 11·3

SATELLITE VII.

Date and G.M.T.				Exposure.	Apparent R.A.			Apparent Dec.			Sat. VII—Jupiter. R.A.		Sat. VII—Jupiter. Dec.	
1907.	d	h	m	min.	h	m	s	°	′	″	m	s	′	″
Dec.	5	16	1·7	140	9	1	56·44	+17	54	46·7	−2	56·39	+34	5·3
,,	10	15	7·3	66	9	1	14·38	17	56	27·6	3	9·74	32	35·0
,,	14	14	48·9	120	9	0	28·40	17	58	24·7	3	18·68	30	58·9
1908.														
Jan.	3	13	11·6	120	8	54	18·91	18	14	29·6	3	29·30	17	33·3
,,	11	13	10·6	90	8	51	3·83	18	22	51·1	3	11·35	+ 9	49·0
,,	27	12	41·4	65	8	44	9·55	18	40	54·4	1	50·71	− 7	25·1
Feb.	1	11	52·1	112	8	42	3·27	18	46	46·4	1	15·31	12	31·8
,,	3	10	27·3	80	8	41	14·97	18	49	10·9	1	0·67	14	19·6
,,	3	11	28·4	25	8	41	13·65	18	49	12·2	−1	0·61	14	23·9
,,	22	10	56·6	100	8	34	13·10	19	13	36·7	+1	25·96	26	11·9
,,	23	8	33·8	42	8	33	55·86	19	14	49·7	1	32·08	26	24·3
,,	24	12	24·9	80	8	33	34·27	19	16	22·5	1	39·94	26	38·7
,,	27	10	59·4	70	8	32	41·67	19	20	18·0	1	58·61	27	0·5
,,	28	11	28·6	80	8	32	24·20	19	21	37·4	2	4·67	27	5·1
,,	29	9	39·8	50	8	32	8·79	19	22	50·9	2	10·04	27	5·7
Mar.	21	9	48·7	120	8	28	12·66	19	46	39·3	3	24·59	20	56·8
,,	23	8	59·0	120	8	28	3·35	19	48	19·8	3	27·27	19	52·9
,,	27	8	54·3	43	8	27	51·66	19	51	12·5	3	30·57	17	41·6
,,	28	9	11·1	180	8	27	50·17	19	51	49·8	3	30·86	17	7·7
,,	31	9	16·4	165	8	27	50·10	19	53	30·2	3	31·30	15	20·9
Apr.	24	9	43·5	100	8	31	23·31	+19	53	8·3	+ 3	0·90	− 0	29·4

SATELLITE VIII.

Date and G.M.T.				Exposure.	Apparent R.A.			Apparent Dec.			Sat. VIII—Jupiter. R.A.		Sat. VIII—Jupiter. Dec.	
1908.	d	h	m	min.	h	m	s	°	′	″	m	s	′	″
Jan.	27	12	41·4	65	8	45	51·86	+18	5	1·4	− 0	8·40	−43	18·1
Feb.	1	11	52·1	112	8	43	20·67	18	17	36·3	+ 0	2·08	41	41·8
,,	3	10	27·3	80	8	42	21·90	18	22	25·7	0	6·26	41	4·9
,,	3	11	28·4	25	8	42	20·52	18	22	34·7	0	6·26	41	1·3
,,	22	10	56·6	100	8	33	35·41	19	5	49·8	0	48·27	33	58·9
,,	23	8	33·8	42	8	33	13·98	19	7	37·6	0	50·20	33	36·4
,,	24	12	24·9	80	8	32	47·37	19	9	51·3	0	53·04	33	9·9
,,	27	10	59·4	70	8	31	42·89	19	15	18·1	0	59·83	32	0·4
,,	28	11	28·6	80	8	31	21·76	19	17	6·7	1	2·23	31	35·9
Mar.	27	8	54·3	43	8	26	32·33	19	47	53·5	2	11·24	21	0·6
,,	31	9	16·4	165	8	26	40·24	19	49	12·3	2	21·44	19	38·8
Apr.	3	9	51·7	90	8	26	54·72	19	49	38·8	2	29·22	18	40·9
,,	24	9	43·5	100	8	31	43·33	+19	41	8·9	+3	20·92	−12	28·8

Errors of Tabular Place of Jupiter.

The positions of the satellites given above depend on the positions of certain stars taken from catalogues of the Astronomische Gesellschaft, and will be affected by the mean error in the places of these stars. To eliminate this it is necessary to determine the position of Jupiter referred to the same stars.

For this purpose a number of photographs of Jupiter were taken with the 26-inch refractor, using the occulting shutter. By this means good measurable images of Jupiter were obtained, together with sufficiently exposed images of the reference stars. The field of the 26-inch on a 16 cm. plate is one square degree, and six comparison stars in this area were measured together with Jupiter. The adopted places of these six stars were deduced from measures made on the Astrographic reference plates, the constants of the reference plates being, as mentioned above, derived from all the available stars in the Astronomische Gesellschaft catalogues. The places of the six stars, and therefore the deduced place of Jupiter, will be affected by the mean systematic error of all the catalogue stars on the reference plates, and will thus be comparable with the deduced places of the satellites.

Four photographs were selected for measurement. From two to four images of Jupiter and of each of the stars were measured on each plate in the Astrographic micrometer, and the following are the results obtained :—

Errors of R.A. Tabular Place of Jupiter.

	Tab.—Obs.	
	R.A.	Dec.
	s	"
Jan. 27	−·10	−0·6
Feb. 10	−·14	−0·8
,, 18	−·12	−0·6
May 6	−·08	−0·9
Mean	−·110	−0·72

Corrections for these mean errors have been applied to the tabular places of Jupiter, as already stated.

Royal Observatory, Greenwich:
1908 *June* 10.

Observations of the Satellite of Neptune, from photographs taken at the Royal Observatory, Greenwich, between 1907 *December* 10 *and* 1908 *March* 19.

(*Communicated by the Astronomer Royal.*)

The following measures of position-angle and distance of Neptune's satellite were made from photographs taken with the 26-inch refractor of the Thompson equatorial. The occulting shutter was used as in previous years. The photographs were taken by Messrs. Davidson, Edney, or Melotte, and were measured in a position-micrometer in direct and reversed positions by Messrs. Davidson and Melotte. The tabular positions with which comparison is made were computed from data given in the *Connaissance des Temps*, based on Dr. Hermann Struve's elements, the eccentricity of the orbit being neglected. A discussion of these residuals gives the following differences from Dr. Hermann Struve's elements in the sense Tabular—Observed:

$$du = -0^{\circ}\cdot69, \quad dN = -0^{\circ}\cdot54, \quad dI = +0^{\circ}\cdot11, \quad da = +0''\cdot244$$

giving for the epoch 1908·2

$$a = 16''\cdot027, \quad N = 188^{\circ}\cdot38, \quad I = 116^{\circ}\cdot24$$

Neptune and Satellite.

Position-angle and Distance from photographs taken with the 26-inch Refractor.

Date and G.M.T.					Position-angle.			Distance.		
					Obs.	Tab.	T—O.	Obs.	Tab.	T—O.
1907.	d	h	m	s	°	°	°	″	″	″
Dec.	10	13	47	48	241·68	243·09	+1·41	13·82	14·00	+0·18
	14	12	49	34	341·21	339·80	−1·41	12·04	11·93	−0·11
	14	13	15	59	338·86	338·36	−0·50	11·88	12·02	+0·14
1908.										
Jan.	3	11	35	18(*a*)	217·90	215·22	(−2·68)	11·16	11·86	(+0·70)
	4	11	13	40(*b*)	132·17	134·64	+2·47	13·35	13·97	+0·62
	4	11	39	10	133·26	133·58	+0·32	13·75	14·09	+0·34
	9	11	58	11	202·98	203·38	+0·40	10·87	11·37	+0·50
	11	11	27	45	82·99	83·75	+0·76	16·36	16·28	−0·08
	11	11	53	29	82·24	82·99	+0·75	16·30	16·22	−0·08
	27	9	49	5	179·03	178·08	−0·95	10·70	11·20	+0·50
	27	10	13	39	175·19	176·50	+1·31	11·34	11·23	−0·11
Feb.	3	9	15	21(*c*)	108·18	108·29	+0·11	16·25	16·42	+0·17
	6	10	2	5(*d*)	285·08	284·32	−0·76	16·23	16·62	+0 39
	6	10	35	58(*c*)	282·40	283·37	+0·97	16·29	16·67	+0·38
	10	10	0	38(*e*)	53·51	54·88	+1·37	13·08	13·35	+0·27

Position-angle and Distance—continued.

Date and G.M.T.				Position-angle.			Distance.		
				Obs.	Tab.	T—O.	Obs.	Tab.	T—O.
1908.	d	h	m	s	°	°	°	"	"	"
Feb.	10	10	23	22	52·81	53·86	+1·05	13·33	13·25	−0·08
	10	10	43	6	51·44	53·00	+1·56	13·16	13·17	+0·01
	18	9	18	23	275·88	275·48	−0·40	16·50	16·72	+0·22
	18	9	41	43	274·99	274·83	−0·16	16·66	16·72	+0·06
	18	10	5	24	272·87	274·15	+1·28	16·31	16·71	+0·40
Mar.	5	8	1	6	23·12	23·92	+0·80	11·20	11·24	+0·04
	5	8	33	32(*f*)	19·93	21·95	+2·02	11·11	11·18	+0·07
	14	9	9	29(*b*)	180·41	182·81	+2·40	10·55	10·91	+0·36
	14	9	49	10	177·86	180·31	+2·45	10·61	10·93	+0·32
	19	10	39	41	243·84	245·23	+1·39	13·97	14·21	+0·24
	19	11	4	17	243·49	244·28	+0·79	14·06	14·11	+0·05

(*a*) Very poor photograph.
(*b*) Very diffused.
(*c*) Very faint.
(*d*) Poor photograph, satellite faint.
(*e*) Satellite diffused.
(*f*) Satellite faint.

The satellite of Neptune has now been regularly observed at Greenwich during seven successive oppositions. The differences from Dr. Hermann Struve's elements found from the observations are as follows:—

Epoch.	No. of plates.	*du*.	*dN*.	*dI*.	*da*.	2 *e*. sin Q.	2 *e*. cos Q.
		°	°	°	"		
1902·1	51	−0·95	−0·42	−0·12	+0·087	+·0005	−·0004
1903·1	63	−0·67	−0·40	−0·30	+0·092	+·0005	+·0016
1904·1	51	−0·83	−0·72	−0·20	+0·033	−·0017	+·0002
1905·1	57	−0·82	−0·95	−0·37	+0·069	+·0038	+·0053
1906·1	59	−0·90	−1·17	+0·12	+0·085	−·0043	−·0027
1907·2	29	−1·02	−0·93	−0·20	−0·008	+·0024	+·0009
1908·2	25	−0·64	−0·44	+0·11	+0·244	−·0110	+·0032

The very small eccentricity has been treated as a quantity to be determined in these solutions; accordingly the figures for *du*, *dN*, *dI*, and *da* differ slightly from those published previously in the *Monthly Notices*, in which the eccentricity is taken as 0.

Royal Observatory, Greenwich:
1908 *June* 11.

On the Mathematical Theory of Two Star-drifts, and on the Systematic Motions of Zodiacal Stars.

By A. S. Eddington, B.A., M.Sc.

In a former paper,* the distribution of the proper motions of the stars was discussed from the point of view of the hypothesis of two drifts of stars. The method of analysis consisted in finding by trial and error the constants of two drifts which would lead to a distribution of proper motions (as regards direction) agreeing as nearly as possible with the observed distribution. There are evident disadvantages in a method of this kind; the difficulty of simultaneously adjusting five disposable constants, so as to obtain the best agreement with observation, is considerable; there is room for bias in deciding what is the best agreement with observation; and the method is inapplicable to the discussion of a series of proper motions of stars sparsely distributed over a large region of the sky, such as the Bradley proper motions.

It is the object of the first part of this paper (*Mathematical Theory*) to obtain from the observational data direct equations to determine the constants of the drifts. In the second part the theory is applied to the analysis of the proper motions of the Catalogue of Zodiacal Stars (*Astronomical Papers of the American Ephemeris*, vol. viii., part iii.); this is an example of a series of proper motions which, while sufficiently numerous to afford a good determination of the velocities of the drifts, are much too sparsely distributed to be analysed by the method of trial and error.

Mathematical Theory.

I have defined a "drift" to be a system of stars in which the motions of the individual stars relative to the mean of the system are haphazard; the whole system has in general a drift-motion relative to the Sun. For the sake of precision, the distribution of the haphazard or "peculiar motions" in the ideal drift is taken to be according to Maxwell's law.

In "Systematic Motions," p. 36, the law of distribution of the proper motions in the different directions was discussed for a system of stars forming a drift thus defined; the analysis may be briefly recapitulated here.

A small region of the sky is considered, and motions in the line of sight are neglected, so that the problem is two-dimensional.

Let the number of stars having component peculiar (linear) velocities between the limits (u, v) and $(u+du, v+dv)$ be

$$\frac{n_1 h^2}{\pi} e^{-h^2(u^2+v^2)} du dv \quad . \quad . \quad . \quad . \quad (1)$$

in accordance with Maxwell's law.

* "The Systematic Motions of the Stars," *Monthly Notices*, vol. lxvii. p. 34.

n_1 is the total number of stars of the drift.
h a constant inversely proportional to the mean peculiar speed of the stars.

Let v_1 be the velocity of the drift.
θ_1 the inclination of the direction of this drift-velocity to the axis of x.
r, θ the amount and direction of the resultant linear velocity of a star (*i.e.* drift velocity and peculiar velocity compounded).
$\rho\, d\theta$ the number of stars having proper motions in directions inclined to Ox between θ and $\theta + d\theta$.

Then $$u^2 + v^2 = r^2 + v_1^2 - 2v_1 r \cos(\theta - \theta_1)$$

$$du\,dv = r\,dr\,d\theta$$

and hence $$\rho d\theta = \frac{n_1 h^2}{\pi} d\theta \int_0^\infty e^{-h^2\{r^2 + v_1^2 - 2v_1 r \cos(\theta - \theta_1)\}} r\,dr \quad . \quad (2)$$

By means of the substitution

$$x = hr - hv_1 \cos(\theta - \theta_1)$$

this expression can easily be reduced to

$$\rho = \frac{n_1 h^2}{\pi} e^{-h^2 v_1^2} \left\{ \tfrac{1}{2} + \tau e^{\tau^2} \int_{-\tau}^{\infty} e^{-x^2} dx \right\} \quad . \quad . \quad (3)$$

where $$\tau = hv_1 \cos(\theta - \theta_1)$$

or setting $$f(\tau) = \tfrac{1}{2} + \tau e^{\tau^2} \int_{-\tau}^{\infty} e^{-x^2} dx$$

$$\rho = \frac{n_1}{\pi} e^{-h^2 v_1^2} f\big(hv_1 \cos(\theta - \theta_1)\big) \quad . \quad . \quad . \quad (4)$$

The function f was tabulated in "Systematic Motions," p. 36; a factor $\frac{2}{\sqrt{\pi}}$ was, however, inserted in defining it there, which it has been more convenient to drop now.

Consider now a series of proper motions of stars belonging to two drifts; distinguishing the corresponding constants by suffixes 1 and 2, we shall have

$$\rho = \frac{n_1}{\pi} e^{-h^2 v_1^2} f(hv_1 \cos\theta - \overline{\theta_1}) + \frac{n_2}{\pi} e^{-h^2 v_2^2} f(hv_2 \cos\theta - \overline{\theta_2}) \quad . \quad (5)$$

If the observed proper motions are considered to correspond to two drifts, a series of equations of condition will be obtained by giving various values to θ in (5), the corresponding values of ρ being derived from observation. From these the six unknowns, n_1, n_2, v_1, v_2, θ_1, θ_2 are to be derived; these, however, may be at

once reduced to five unknowns, for $n_1 + n_2$ is equal to the total number of stars.

It is of course impossible, with non-linear equations of condition such as these, rigorously to determine the values of the constants which shall make the sum of the squares of the residuals a minimum. In order to obtain what may serve, as it were, for normal equations, I have expanded each side of equation (5) in a Fourier series, and have equated the first few coefficients on each side until sufficient equations were thus formed. It is clear that, at least if the observed distribution really does correspond to two drifts, a good agreement between the observed and theoretical distributions will have been obtained when the Fourier series representing them agree as far as the coefficients of $\cos 3\theta$ and $\sin 3\theta$, and it is not likely that the agreement could be much further improved. Discordances represented by differences in the coefficients of higher harmonics are likely to be of an accidental and unimportant character.

In practice, the process of equating Fourier coefficients is equivalent to equating the sums $\Sigma \cos\theta$, $\Sigma \sin\theta$, $\Sigma \cos 2\theta$, $\Sigma \sin 2\theta$, etc., for the observed proper motions, to the corresponding integrals for the theoretical distributions. These integrals are evaluated in the note at the end of this paper. We find there that for a single drift $(n_1, v_1\ \theta_1)$,

$$\left.\begin{aligned}
\int_0^{2\pi} \rho \cos(\theta - \theta_1) d\theta &= n_1 \frac{\sqrt{\pi}}{2} h v_1 e^{-\frac{1}{4}h^2 v_1^2} \left\{ J_0(\iota \tfrac{1}{2} h^2 v_1^2) - \iota J_1(\iota \tfrac{1}{2} h^2 v_1^2) \right\} \\
&= n_1 \,|\, C(hv_1) \,|\quad \text{say} \\
\int_0^{2\pi} \rho \cos 2(\theta - \theta_1) d\theta &= n_1 \left(1 - \frac{1 - e^{-h^2 v_1^2}}{h^2 v_1^2} \right) = n_1 \,|\, D(hv_1) \,|\quad \text{say} \\
\int_0^{2\pi} \rho \cos 3(\theta - \theta_1) d\theta &= n_1 \frac{\sqrt{\pi}}{2} h v_1 e^{-\frac{1}{4}h^2 v_1^2} \left\{ J_0(\iota \tfrac{1}{2} h^2 v_1^2) - \iota J_1(\iota \tfrac{1}{2} h^2 v_1^2) \left(1 - \frac{4}{h^2 v_1^2} \right) \right\} \\
&= n_1 \,|\, E(hv_1) \,|\quad \text{say}
\end{aligned}\right\} \quad (6)$$

and evidently from the symmetry of the drift

$$\int_0^{2\pi} \rho \sin(\theta - \theta_1) d\theta = \int_0^{2\pi} \rho \sin 2(\theta - \theta_1) d\theta = \int_0^{2\pi} \rho \sin 3(\theta - \theta_1) d\theta = 0 \quad . \quad . \quad (7)$$

A table of the functions here introduced is given later (see Table I.).

It is here convenient to introduce complex quantities into the analysis, which will be denoted always by capital letters.

Let

$$\left.\begin{aligned}
V_1 &= v_1 e^{\iota\theta_1} \\
C_1 &= C(hV_1) = |\, C(hv_1) \,|\, e^{\iota\theta_1} \\
D_1 &= D(hV_1) = |\, D(hv_1) \,|\, e^{2\iota\theta_1} \\
E_1 &= E(hV_1) = |\, E(hv_1) \,|\, e^{3\iota\theta_1}
\end{aligned}\right\} \quad . \quad . \quad . \quad (8)$$

The moduli have already been defined by equations (6).

Still considering one drift only

$$\int_0^{2\pi} \rho e^{\iota\theta} d\theta = e^{\iota\theta_1} \int_0^{2\pi} \rho e^{\iota(\theta-\theta_1)} d\theta$$

$$= e^{\iota\theta_1} \int_0^{2\pi} \rho \cos(\theta-\theta_1) d\theta + \iota e^{\iota\theta_1} \int_0^{2\pi} \rho \sin(\theta-\theta_1) d\theta$$

$$= e^{\iota\theta_1} n_1 |\, C(hv_1) \,|\,.$$

Thus
$$\int_0^{2\pi} \rho e^{\iota\theta} d\theta = n_1 C_1$$

Similarly
$$\int_0^{2\pi} \rho e^{2\iota\theta} d\theta = n_1 D_1$$
$$\int_0^{2\pi} \rho e^{3\iota\theta} d\theta = n_1 E_1 \qquad \ldots\ldots (9)$$

Return now to the consideration of two drifts. Knowing for every star the apparent direction θ (measured from an arbitrary initial line) of its observed motion, we can calculate $\Sigma \cos\theta$, $\Sigma \sin\theta$, $\Sigma \cos 2\theta$, etc. for all the stars.

Let
$$L = \frac{1}{n} \Sigma e^{\iota\theta} = \frac{1}{n} \left(\Sigma \cos\theta + \iota \Sigma \sin\theta \right)$$

$$M = \frac{1}{n} \Sigma e^{2\iota\theta} = \frac{1}{n} \left(\Sigma \cos 2\theta + \iota \Sigma \sin 2\theta \right)$$

$$N = \frac{1}{n} \Sigma e^{3\iota\theta} = \frac{1}{n} \left(\Sigma \cos 3\theta + \iota \Sigma \sin 3\theta \right)$$

where n is the total number of stars discussed.

And let $\frac{n_1}{n} = \frac{1}{2}(1+\alpha)$

so that $\frac{n_2}{n} = \frac{1}{2}(1-\alpha)$ since $n = n_1 + n_2$.

The equation of the observed and theoretical values of $\Sigma e^{\iota\theta}$, $\Sigma e^{2\iota\theta}$, and $\Sigma e^{3\iota\theta}$, in the case of two drifts, therefore leads to

$$\left.\begin{aligned} 2L &= (1+\alpha) C_1 + (1-\alpha) C_2 \\ 2M &= (1+\alpha) D_1 + (1-\alpha) D_2 \\ 2N &= (1+\alpha) E_1 + (1-\alpha) E_2 \end{aligned}\right\} \qquad \ldots\ldots (10)$$

To solve these equations write

$$\left.\begin{aligned} C &= \zeta P \\ D &= \zeta P^2 \\ E &= \frac{1}{\gamma} \zeta P^3 \end{aligned}\right\} \qquad \ldots\ldots (11)$$

Since the arguments of C_1, D_1, and E_1 are θ_1, $2\theta_1$, and $3\theta_1$, ζ and γ are real, and the argument of P_1 is θ_1. Further, it is found that

for the range of values of hV (up to about $|hV| = 1{\cdot}8$) occurring in the actual drifts, γ may be assumed constant and equal to $1{\cdot}163$. In Table I. are given the values of ζ and the moduli of C, D, E, δE and P for various values of hV. The values of $\delta E = E - \frac{1}{1{\cdot}163}\zeta P^3$ are given in order to show the accuracy of the approximation made.

TABLE I.

hV.	P.	ζ.	C.	D.	E.	δE.
0·0	·0000	1·585	·0000	·0000	·0000	·000
0·1	·0566	1·573	·0884	·0050	·0002	·000
0·2	·1124	1·561	·1755	·0197	·0017	·000
0·3	·1680	1·548	·2600	·0437	·0057	− ·001
0·4	·2227	1·531	·3409	·0759	·0134	·001
0·5	·2762	1·510	·4171	·1152	·0253	·002
0·6	·3284	1·486	·4879	·1602	·0420	·003
0·7	·3789	1·459	·5528	·2094		·005
0·8	·4275	1·431	·6115	·2614	·0901	·006
0·9	·4739	1·401	·6640	·3147		·007
1·0	·5179	1·371	·7103	·3679	·1558	·008
1·1	·5595	1·342	·7507	·4200		·008
1·2	·5984	1·313	·7856	·4701	·2337	·008
1·3	·6345	1·285	·8156	·5175		·007
1·4	·6678	1·259	·8410	·5617	·3172	·005
1·5	·6984	1·235	·8626	·6024	·3590	− ·003
1·6	·7262	1·213	·8807	·6396	·4000	·000
1·7	·7514	1·192	·8959	·6732		+ ·004
1·8	·7742	1·174	·9087	·7035	·4774	·009
1·9	·7946	1·157	·9194	·7305		·014
2·0	·8128	1·142	·9284	·7546	·5468	+ ·019

With this substitution, (10) becomes

$$2L = (1+\alpha)\,\zeta_1 P_1 + (1-\alpha)\,\zeta_2 P_2\,;\ \text{etc.}$$

Now let

$$\left.\begin{aligned}(1+\alpha)\zeta_1 &= (1+\beta)k\\ (1-\alpha)\zeta_2 &= (1-\beta)k\end{aligned}\right\} \quad . \quad . \quad . \quad (12)$$

so that

$$k = \tfrac{1}{2}(1+\alpha)\zeta_1 + \tfrac{1}{2}(1-\alpha)\zeta_2 \quad . \quad . \quad . \quad (13)$$

We obtain finally

$$2L/k = (1+\beta)P_1 + (1-\beta)P_2 \quad . \quad . \quad . \quad (14)$$

$$2M/k = (1+\beta)P_1^2 + (1-\beta)P_2^2 \quad . \quad . \quad . \quad (15)$$

$$2N\gamma/k = (1+\beta)P_1^3 + (1-\beta)P_2^3 \quad . \quad . \quad . \quad (16)$$

It will be seen from (13) that k is a weighted mean between ζ_1 and ζ_2, the weights being proportional to the number of stars in the two drifts. Now looking at Table I. we see that ζ_1 and ζ_2 vary within fairly narrow limits; actually the values of hv_1 and hv_2 in the different parts of the sky are such that, except in a few unusual cases, k will lie between 1·35 and 1·45. For a first approximation, we may either calculate a provisional value of k from the roughly known speeds of the drifts, or we may assume for it the value 1·40, which will certainly be not far from the truth. A second approximation may afterwards be made if desired, using the value of k deduced from the results of the first approximation; in practice, however, this is usually quite unnecessary.

Thus k may be assumed to be known, and the solution of (14), (15), and (16) is then easy. We find

$$\frac{4}{k^2}(N\gamma k - LM) = (1-\beta^2)(P_1-P_2)^2(P_1+P_2) \quad . \quad (17)$$

$$\frac{4}{k^2}(Mk - L^2) \quad = (1-\beta^2)(P_1-P_2)^2 \quad . \quad . \quad . \quad (18)$$

therefore $$\frac{N\gamma k - LM}{Mk - L^2} = P_1 + P_2 = 2K \quad \text{say} \quad . \quad . \quad (19)$$

whence solving (14) and (19) for P_1 and P_2

$$\left.\begin{aligned} P_1 &= K + \left(\frac{L}{k} - K\right)\Big/\beta \\ P_2 &= K - \left(\frac{L}{k} - K\right)\Big/\beta \end{aligned}\right\} \quad . \quad . \quad . \quad (20)$$

and substituting these values in (15) we easily find

$$\beta = \frac{L - Kk}{\sqrt{(L-Kk)^2 + (Mk - L^2)}} \quad . \quad . \quad . \quad (21)$$

We also find from (14) and (18)

$$\left.\begin{aligned} P_1 &= \frac{1}{k}\left(L + \sqrt{\frac{1-\beta}{1+\beta}}\sqrt{Mk - L^2}\right) \\ P_2 &= \frac{1}{k}\left(L - \sqrt{\frac{1+\beta}{1-\beta}}\sqrt{Mk - L^2}\right) \end{aligned}\right\} \quad . \quad . \quad (22)$$

Equations (21) and (22) constitute the solution of the problem.

The argument of P_1 is θ_1, and hv_1 can be found from the modulus by means of Table I. ζ_1 and ζ_2 can be taken from the same table, and then α is found from the formula

$$\frac{1-\alpha}{1+\alpha} = \frac{1-\beta}{1+\beta}\cdot\frac{\zeta_1}{\zeta_2}.$$

In the practical application of these formulæ a difficulty arises from the fact that, whereas β is by definition a real quantity, the right-hand side of (21), which is derived from observation, is in general a complex quantity.

The reason is this: equations (14), (15), and (16) practically constitute six equations, for each of them has a real and an imaginary part, which must be satisfied separately. But there are only five unknowns. In the above solution the equations have been reconciled by introducing a sixth fictitious unknown, viz. the imaginary part of α. If, however, α and β are to be purely real, the six equations cannot be exactly satisfied. Now it is clear that, just as we were justified in rejecting equations derived from equating the higher harmonics, we should attach more weight to equations (14) and (15) than to (16). It seems reasonable to satisfy the former exactly, and to throw the discordance entirely on (16). In other words, the observed and theoretical distributions will be made to agree exactly as regards coefficients of $\sin\theta$, $\cos\theta$, $\sin 2\theta$, $\cos 2\theta$, in the corresponding Fourier series, the coefficients of $\sin 3\theta$ and $\cos 3\theta$ will be made to agree as well as possible, and all higher harmonics will, as before, be entirely disregarded. If this rule seems rather arbitrary, it must be remembered that the whole question is one of weighting the equations of condition, that the ideal system of weighting is impracticable, and that we are seeking a practicable system which shall not unduly waste the material afforded by observation.

As (16) is not to be exactly satisfied, our solution must be chosen to make the sum of the squares of the two residuals (of the real and imaginary parts of the equation) a minimum. This is equivalent to making a minimum the modulus of the residual of the complex equation. Or we may satisfy the equation exactly by replacing N by $N+\delta N$, where $|\delta N|$ is to be a minimum.

Following this change through the solution previously given, let the alteration in N change $L-Kk$ to $(L-Kk)_0$; we see from (19) that if $|\delta N|$ is a minimum,

$$|(L-Kk)-(L-Kk)_0| \text{ will be a minimum.}$$

But (21) may be written

$$\frac{1}{\beta^2}=1+\left\{\frac{\sqrt{Mk-L^2}}{L-Kk}\right\}^2;$$

therefore if β is real, we must choose $(L-Kk)_0$ so that its argument is the same as, or differs by 180° from, that of $\sqrt{Mk-L^2}$.

(The alternative $\arg(L-Kk)_0=\arg\sqrt{Mk-L^2}\pm 90^\circ$ leads to a value of β greater than unity, which must evidently be excluded.)

The conditions are satisfied if $(L-Kk)_0$ is the projection of the vector $L-Kk$ (in an Argand diagram) on the direction of the vector $\pm\sqrt{Mk-L^2}$; for this makes $|L-Kk-(L-Kk)_0|$ a minimum, subject to β being real.

Therefore

$$|(L - Kk)_0| = |L - Kk| \cos\{\arg(L - Kk) - \arg\sqrt{Mk - L^2}\} \quad (23)$$

and

$$\frac{1}{\beta^2} = 1 + \frac{|Mk - L^2|}{|(L - Kk)_0|^2} \quad . \quad . \quad . \quad (24)$$

and P_1 and P_2 are given by (22) without modification.

The sign of β is positive or negative according as $(L - Kk)_0$ is of the same or opposite sign to $\sqrt{Mk - L^2}$. Either square root of $Mk - L^2$ may be taken, but the same root must be adhered to throughout. It is, however, desirable to choose the root whose argument agrees roughly with the direction of motion of Drift I relative to Drift II, otherwise the suffixes of the constants will not agree with the usual designations of the drifts.

Example.

The following example shows the practical application of the method. It refers to Region C (R.A. 2^h–6^h, N.P.D. 20°–52°) of the Groombridge proper motions.

From the observed proper motions I find

Total number of stars $n = 512$.

$$\Sigma \cos\theta = +73{\cdot}0 \qquad \Sigma \sin\theta = +219{\cdot}0$$
$$\Sigma \cos 2\theta = -42{\cdot}3 \qquad \Sigma \sin 2\theta = +124{\cdot}1$$
$$\Sigma \cos 3\theta = -98{\cdot}45 \qquad \Sigma \sin 3\theta = +23{\cdot}1$$

hence, dividing by n,

$$L = +{\cdot}143 + {\cdot}428\,\iota$$
$$M = -{\cdot}083 + {\cdot}242\,\iota$$
$$N = -{\cdot}192 + {\cdot}045\,\iota$$
$$\text{and} \quad \gamma N = -{\cdot}223 + {\cdot}052\,\iota \quad (\gamma = 1{\cdot}163).$$

Assume provisionally $k = 1{\cdot}40$, we find

$$N\gamma k - LM = -{\cdot}1967 + {\cdot}0737\iota = {\cdot}2100 \text{ expi}^* \ 159°{\cdot}5$$
$$Mk - L^2 = +{\cdot}0466 + {\cdot}2164\iota = {\cdot}2214 \text{ expi} \ 77°{\cdot}9$$

therefore by division (equation (19))

$$2K = {\cdot}948 \text{ expi } 81°{\cdot}6$$
$$K = +{\cdot}069 + {\cdot}469\,\iota$$

therefore

$$L - Kk = +{\cdot}046 - {\cdot}229\,\iota$$
$$= {\cdot}234 \text{ expi } 281°{\cdot}4;$$

also

$$\sqrt{Mk - L^2} = {\cdot}470 \text{ expi } 39°{\cdot}0.$$

If we continue the solution with $L - Kk$ unmodified, equation (21) gives

$$\beta = -{\cdot}300 - {\cdot}442\,\iota.$$

* Expi $x = e^{\iota x}$.

But modifying it in accordance with (23)

$$(L - Kk)_0 = \cdot 234 \cos(281^\circ\cdot 4 - 39^\circ\cdot 0) \operatorname{expi} 39^\circ\cdot 0$$
$$= -\cdot 1083 \operatorname{expi} 39^\circ\cdot 0$$

and $$\frac{\sqrt{Mk - L^2}}{(L - Kk)_0} = -4\cdot 53,$$

whence $$\beta = -\frac{1}{\sqrt{1 + (4\cdot 53)^2}} = -\cdot 215.$$

This value of β must now be substituted in (22),

$$\sqrt{\frac{1-\beta}{1+\beta}} = 1\cdot 244.$$

$$\sqrt{\frac{1-\beta}{1+\beta}}\sqrt{Mk - L^2} = \cdot 610 \operatorname{expi} 39^\circ\cdot 0$$
$$= \cdot 474 + \cdot 384\,\iota$$

and $$L = \cdot 143 + \cdot 428\,\iota.$$

Sum $$= kP_1 = \cdot 617 + \cdot 812\,\iota,$$
$$= 1\cdot 020 \operatorname{expi} 52^\circ\cdot 8.$$

Therefore $$P_1 = \cdot 729 \operatorname{expi} 52^\circ\cdot 8.$$

Similarly we find $$P_2 = \cdot 174 \operatorname{expi} 132^\circ\cdot 7.$$

The directions of the two drifts are accordingly

$$\theta_1 = 52^\circ\cdot 8 \qquad \theta_2 = 132^\circ\cdot 7.$$

To find their velocities we make use of Table I., which gives the corresponding values of $|P|$ and hv,

$$hv_1 = 1\cdot 61 \qquad hv_2 = 0\cdot 31;$$

also from Table I.,

$$\zeta_1 = 1\cdot 211 \qquad \zeta_2 = 1\cdot 547$$

but $$\frac{1-\alpha}{1+\alpha} = \frac{1-\beta}{1+\beta}\frac{\zeta_1}{\zeta_2}$$
$$= 1\cdot 548 \times \frac{1\cdot 211}{1\cdot 547}$$
$$= 1\cdot 212,$$

whence $$\alpha = -\cdot 096$$

$$\tfrac{1}{2}(1+\alpha) = \cdot 452 \quad \tfrac{1}{2}(1-\alpha) = \cdot 548.$$

Thus 45·2 per cent. of the stars belong to Drift I and 54·8 per cent. to Drift II.

Finally, we can check the provisional value of k,

$$k = \tfrac{1}{2}(1+\alpha)\zeta_1 + \tfrac{1}{2}(1-\alpha)\zeta_2 = 1\cdot 395,$$

agreeing with the adopted value 1·40, so that no further approximation is needed.

The results obtained by this method agree closely with those previously obtained by the method of trial and error ("Systematic Motions," p. 50). I give below a comparison of the constants determined in the two ways for four of the Groombridge Regions; it affords some sort of indication of the reliability of both methods.

		Analytical Method.		Trial and Error.	
		Drift I.	Drift II.	Drift I.	Drift II.
Region A	$h\nu$	1·38	0·31	1·65	0·30
	θ	0°	157°	0°	155°
	n*	52·5	47·5	48·5	51·5
Region B	$h\nu$	1·47	0·48	1·55	0·45
	θ	12°	131°	10°	130°
	n	49·5	50·5	49·5	50·5
Region C	$h\nu$	1·61	0·31	1·65	0·30
	θ	53°	133°	55°	125°
	n	45·2	54·8	41·5	58·5
Region F	$h\nu$	0·86	0·77	1·20	0·45
	θ	226°	75°	225°	80°
	n	63·0	37·0	46 5	53·5

* Per cent.

The weakest point in the solution is exemplified in the case of Region F; the determination of α (or β) is not very satisfactory. The difference in the two solutions in the case of Region F depends entirely on the division of the stars between the two drifts; if we had adopted the same value of α in the two cases, the remaining four constants found from (22) would have agreed almost exactly with those found by trial and error. The weakness of the determination of β can be shown analytically.

Differentiating (21),

$$\frac{d\beta}{dN} = -\tfrac{1}{2}k^2\gamma\{(Mk - L^2) + (L - Kk)^2\}^{-\frac{3}{2}}$$

$$= -\frac{4\gamma}{k}(P_1 - P_2)^{-3}.$$

Now $|P_1 - P_2|$ is generally about 0·7, but may be less, hence $\left|\frac{d\beta}{dN}\right|$ is generally about 10, but may be more.

Thus in a region containing 500 stars, a change of 2·5 in $\Sigma \cos 3\theta$ or $\Sigma \sin 3\theta$ would produce a change in N of ·005, and the corresponding change in β or α might be $\pm$ ·05.

The question arises whether the weakness of the determination of β is a defect of the method of analysis or is necessarily involved in the nature of the observational data. I think there is little doubt that the latter alternative is correct; I have examined various other methods of determining β, but all are rather insensitive. In the case of Region F, both the solutions given above are found to agree with the observed distributions almost equally well.

Thus it seems likely that even in an ideal solution we should have one equation very much weaker than the other four (the weakness may be more pronounced for some regions of the sky than for others). As the five constants depend on one another and on this fifth equation, they may all share in the uncertainty.

One or two considerations help to avoid this difficulty to some extent. We may be content to assume $\alpha = 0$, *i.e.* that the stars are evenly divided between the two drifts; all evidence seems to indicate that this is approximately true, and it is conceivable that there may be some physical reason for it. Or, instead of adopting the value of α found for the particular region, we may adopt a mean found from all the regions discussed; this will have a much smaller probable error. But the most fortunate circumstance is that *we may determine the relative motion of the two drifts almost independently of* α.

Equation (18) gives

$$P_1 - P_2 = \frac{2}{k}(1 - \beta^2)^{-\frac{1}{2}} \sqrt{Mk - L^2}\,.$$

If β lies anywhere between $-0{\cdot}3$ and $+0{\cdot}3$, $(1 - \beta^2)^{-\frac{1}{2}}$ may be put equal to $\cdot 98$ with an error certainly less than 3 per cent.; but this range of β includes all values likely to occur. Only in the case of a very great disparity in the distribution between the two drifts could a value outside these limits occur. Thus $P_1 - P_2$ is nearly independent of β or α. P is a sufficiently nearly linear function of hV for $h(V_1 - V_2)$, to be also nearly independent of α. Thus although adopting $\alpha = 0$ may lead to some error in the determinations of hV_1 and hV_2, the error will nearly be eliminated from the determination $h(V_1 - V_2)$. This relative motion of the two drifts is the quantity which most interests us, especially as a systematic error in the proper motions does not affect its determination so adversely as it affects the determinations of hV_1 and hV_2.

Systematic Motions of Zodiacal Stars.

I have applied the theory given above to the discussion of the proper motions of the zodiacal stars. The proper motions were taken from the Catalogue of Zodiacal Stars, *Astronomical Papers of the American Ephemeris*, vol. viii., part iii. Excluding the stars of the Pleiades, this contains 1533 proper motions. I divided the zodiac into sixteen regions, each extending $22\frac{1}{2}^\circ$ in longitude by about 16° in latitude; these are denoted successively by I*a*, II*a*, . . . VIII*a*, I*b*, ... VIII*b*, the centre of region I*a* being at the first point of Aries. As regions I*a* and I*b* are diametrically opposite to one another, the observed motions are in parallel planes, and the two regions may be treated together; similarly, the other regions can be treated together in pairs. Thus the number of regions is virtually reduced to eight, each containing from 150 to 250 stars.

The distribution of the proper motions as regards direction in the eight regions is shown in Table II. Opposite $\theta = 0^\circ$ in the

first column is given the number of stars whose observed motions are in directions between $\theta = 355^\circ$ and $\theta = 5^\circ$, and so on. The numbers have not been smoothed. For stars in I*a*, II*a*, etc., $\theta = 0^\circ$ is in the direction of increasing R.A., and $\theta = 90^\circ$ in the direction of increasing Dec. For stars in I*b*, II*b*, etc., the reckoning of θ agrees with that in the opposite parallel planes I*a*, II*a*, etc.

TABLE II.

Distribution of the Proper Motions in Direction.

	Regions.							
θ	I.	II.	III.	IV.	V.	VI.	VII.	VIII.
0°	16	14	9	5	7	2	1	2
10	11	8	3	8	0	0	5	4
20	3	3	3	2	1	1	2	2
30	2	6	0	2	3	4	2	1
40	1	2	2	0	1	2	0	2
50	4	3	0	2	3	2	3	0
60	2	1	0	1	0	1	1	0
70	3	2	0	0	0	0	2	2
80	2	1	0	1	1	0	0	2
90	3	3	1	1	1	0	1	1
100	1	1	1	0	1	1	0	0
110	1	1	0	2	1	1	2	0
120	0	2	1	1	0	4	2	3
130	0	1	1	2	3	2	2	4
140	1	0	1	2	0	3	5	4
150	2	1	0	1	4	1	3	5
160	3	2	1	2	5	6	4	3
170	1	2	3	2	3	2	9	11
180	0	1	2	2	6	10	8	13
190	6	4	1	1	6	7	18	14
200	6	8	4	3	8	9	12	12
210	5	5	6	3	5	9	12	14
220	6	4	3	5	11	16	14	8
230	10	5	6	8	11	20	10	6
240	2	7	2	4	13	14	16	4
250	2	6	3	10	9	17	13	4
260	1	3	6	7	9	9	9	5
270	6	5	2	7	21	12	3	8
280	3	6	9	12	17	10	6	4
290	4	7	13	20	10	8	10	4
300	6	7	11	18	16	8	0	10
310	12	7	8	13	13	6	7	3
320	17	11	15	27	13	6	2	2
330	14	4	12	18	8	4	2	5
340	9	20	18	23	6	1	0	3
350	10	17	9	25	5	0	8	4
Total	175	180	156	240	221	198	194	169

It will be seen, by looking down the columns of the Table that the two streams are plainly marked in Regions I, II, III, and VIII. In the other four regions their directions (projected on the sky) are inclined at an acute angle, and the existence of the two streams is rather concealed. The success of the analysis in these four regions is on that account especially interesting. It may be noticed that this belt of the sky is not so favourable as the Groombridge region for showing conspicuously the separation into two streams; the centre of the latter region lies between the two apices, so that in it the streams are in nearly opposite directions.

Although, in the main, pairs of regions such as I*a* and I*b* were treated together, I thought it safer to calculate $\sqrt{Mk-L^2}$ and L for I*a* and I*b* separately, and to take the mean afterwards. This was in order to avoid the possible effects of systematic error, by ensuring that the difference of motion of the two drifts found from the observations was a difference of motion of intermingled systems of stars, and not the difference in the apparent motion of stars in I*a* from that of stars in I*b*. Actually, however, the precaution might have been omitted. I found that in every region very nearly the same result was obtained whether the two halves were treated separately or together.

For the solutions I used entirely the equations—

$$P_1=\frac{1}{k}\left(L+\sqrt{\frac{1-\beta}{1+\beta}}\sqrt{Mk-L^2}\right)$$

$$P_2=\frac{1}{k}\left(L-\sqrt{\frac{1+\beta}{1-\beta}}\sqrt{Mk-L^2}\right)$$

I did not calculate N, or attempt to find β from the observations. The final results given below depend, therefore, on the assumption (based on previous experience) that the stars are evenly distributed between the two drifts; but calculations are given which show to what extent the results obtained would need to be modified if the assumption is incorrect.

As a preliminary I made two solutions, (*a*) assuming $\sqrt{\frac{1-\beta}{1+\beta}}=1{\cdot}0$ and (*b*) assuming $\sqrt{\frac{1-\beta}{1+\beta}}=1{\cdot}1$; in both cases k was assumed to be 1·40. These correspond to assuming that the stars belonging to Drift I are about (*a*) 55 per cent., (*b*) 50 per cent. of the whole.

The combined results from all the regions were—

Velocity of Drift I	(*a*) 1·61	towards	Latitude	−36°,	Longitude	105°
	(*b*) 1·74	,,	,,	−34°,	,,	106°
Velocity of Drift II	(*a*) 0·64	,,	,,	−42°,	,,	302°
	(*b*) 0·60	,,	,,	−48°,	,,	304°
Velocity of Drift I relative to Drift II	(*a*) 1·84	,,	,,	−16°·3,	,,	109°·4
	(*b*) 1·90	,,	,,	−16°·5,	,,	109°·6

The unit of velocity is $1/h$.

The mutual relative velocity of the two drifts is thus determined nearly independently of the assumed division of the stars between them, a result which has already been arrived at theoretically.

For a final solution, using (b) as a first approximation, I calculated for each region values of k and β, assuming that the stars were equally divided between the drifts.

The values of k for the Regions I, II, ... VIII were respectively—

1·35, 1·35, 1·36, 1·40, 1·44, 1·44, 1·43, 1·38,

and of $\sqrt{\frac{1-\beta}{1+\beta}}$

1·12, 1·12, 1·11, 1·09, 1·07, 1·07, 1·08, 1·10.

Performing the analysis with these values, the constants of the drifts in the eight regions were found as follows:—

Region.	Longitude of Centre.	Drift I.		Drift II.	
		hv_1.	θ_1.	hv_2.	θ_2.
I	0° , 180°	1·57	345°·4	0·46	244°·8
II	22½ , 202½	1·46	350 ·0	0·58	249 ·4
III	45 , 225	1·86	330 ·0	0·63	252 ·2
IV	67½ , 247½	1·68	323 ·5	0·51	271 ·3
V	90 , 270	1·08	299 ·7	0·70	231 ·2
VI	112½ , 292½	1·04	245 ·4	0·54	233 ·3
VII	135 , 315	1·35	208 ·8	0·40	275 ·8
VIII	157½ , 337½	1·49	189 ·2	0·54	292 ·3

If we resolve these drift-velocities along and perpendicular to the plane of the ecliptic, the components perpendicular to the ecliptic derived from each region should agree.

The determinations of these components are—

for Drift I, −·97, −·77, −1·35, −1·19, −·94, −1·00, −·96, −·76, Mean −·99

for Drift II, −·30, −·43, −·52, −·51, −·54, −·48, −·37, −·39, Mean −·44

The accordance of these seems decidedly good when it is remembered that they are derived from regions containing on an average less than 200 stars.

The component drift-velocities in the ecliptic are more or less foreshortened according to the longitude of the region; I determined the mean values by a least-squares solution.

The final results are—

Velocity of Drift I,	1·78	towards	Latitude	−33°·8,	Longitude	105°·5
Velocity of Drift II,	0·59	,,	,,	−47°·9,	,,	303°·7
Velocity of Drift I, relative to Drift II,	1·94	,,	,,	−16°·4,	,,	109°·3

The slight difference between this result and that of solution (*b*) represents the effect of allowing for the variation of β and k from region to region, instead of adopting a mean. The comparison between solutions (*a*) and (*b*) serves to indicate how the results would be changed if the stars were not evenly divided between the drifts. The constants most affected by such a change are the speed of Drift I and the direction of Drift II; the other constants are nearly independent of the assumption, and are therefore more reliably determined.*

From an examination of residuals, I estimate that the probable accidental error of the determination of $k\mathrm{V}_1$ and $k\mathrm{V}_2$ is about $\pm\cdot06$, and the probable errors of the apices of Drifts I and II are respectively about 2° and 6° of a great circle.

Converting latitude and longitude into R.A. and Dec., the positions of the antapices may be compared with previous determinations as follows—

	Drift I.		Drift II.	
Kapteyn . . .	R.A. 85°	Dec. −11°	R.A. 260°	Dec. −48°
Dyson	,, 94°	,, − 7°	,, 240°	,, −74°
Groombridge stars .	,, 90°	,, −19°	,, 292°	,, −58°
Zodiacal stars . .	,, 103°	,, −11°	,, 330°	,, −64°

The great R.A. of the antapices of both drifts found in the present discussion is rather hard to account for.

The velocities of the two drifts $hv_1 = 1\cdot78$, $hv_2 = 0\cdot59$ are in excellent agreement with those found from the Groombridge stars $hv_1 = 1\cdot7$, $hv_2 = 0\cdot5$.

For the velocity of one drift relative to the other, the Zodiacal stars give the value 1·94. From the Groombridge stars (by a least-squares solution from the results of the separate regions) I have found the value 1·90. The determinations of the point towards which this relative velocity is directed (called by Professor Kapteyn the *true* vertex) are

Kapteyn	R.A. 91°	Dec. + 13°
From Groombridge stars .	,, 95°	,, + 3°
,, Zodiacal ,, .	,, 109°	,, + 6°

Professor Schwarzschild's determination of the line of symmetry of motion may be added. This, although based on a rather different theory, is directly comparable with the above. He found,

From Groombridge stars . . R.A. 93° Dec. + 6°

* It should be understood that it is not impossible to determine α from the observations, but simply that when, as in the present case, a few stars are discussed, the value of α is liable to a greater uncertainty than some of the other results. Undoubtedly from the whole 1533 stars a fairly good mean value of α could be determined; but the task of computing it for sixteen regions separately, and taking the mean, would be laborious.

Further, from Professor Dyson's investigation, an R.A. of about 92° for this point may be inferred.

Thus the Right Ascension given by the Zodiacal proper motions is discordant as compared with the other determinations. The number of stars here considered is fewer, and the proper motions are perhaps not so well determined, but I do not think the discordance can be altogether attributed to this. Nor can it be traced to a local anomaly, for it seems to be indicated by the proper motions all round the ecliptic. I have verified by calculation that the Regions I*a* to VIII*a* agree with Regions I*b* to VIII*b* in leading to this high value of the Right Ascension.

Note on the evaluation of certain integrals required in the analysis.

(1) To calculate $n_1 C_1 = \int_0^{2\pi} \rho \cos(\theta - \theta_1) d\theta$, for a single drift.

We may choose the initial line, along the direction of the drift, so that $\theta_1 = 0$.

Then $$\rho = \frac{n_1}{\pi} e^{-h^2 v_1^2} \left\{ \tfrac{1}{2} + \tau e^{\tau^2} \int_{-\tau}^{\infty} e^{-x^2} dx \right\}$$

where $\tau = h v_1 \cos\theta$.

Now $\left\{ \tfrac{1}{2} + \tau e^{\tau^2} \int_{-\tau}^{0} e^{-x^2} dx \right\} \cos\theta$ is an odd function of $\cos\theta$ and vanishes when integrated from 0 to 2π.

Hence $$\int_0^{2\pi} \rho \cos\theta d\theta = \frac{n_1}{\pi} e^{-h^2 v_1^2} \frac{\sqrt{\pi}}{2} \int_0^{2\pi} \tau e^{\tau^2} \cos\theta d\theta$$

$$= \frac{n_1 h v_1}{2\sqrt{\pi}} e^{-h^2 v_1^2} \int_0^{2\pi} \cos^2\theta e^{h^2 v_1^2 \cos^2\theta} d\theta.$$

Now $$\int_0^{2\pi} e^{2\kappa \cos^2\theta} d\theta = e^{\kappa} \int_0^{2\pi} e^{\kappa \cos 2\theta} d\theta$$

$$= 2\pi e^{\kappa} J_0(\iota\kappa).$$

Whence differentiating with respect to κ,

$$2 \int_0^{2\pi} \cos^2\theta e^{2\kappa \cos^2\theta} d\theta = 2\pi e^{\kappa} \left(J_0(\iota\kappa) - \iota J_1(\iota\kappa) \right).$$

Now write $\kappa = \tfrac{1}{2} h^2 v_1^2$ and substitute above

$$C_1 = \frac{\sqrt{\pi}}{2} h v_1 e^{-\frac{1}{2} h^2 v_1^2} \left\{ J_0(\iota \tfrac{1}{2} h^2 v_1^2) - \iota J_1(\iota \tfrac{1}{2} h^2 v_1^2) \right\}.$$

Tables of the Bessel Functions of an imaginary argument are given in *Brit. Assoc. Reports*, 1893 and 1896, and have been used in calculating Table I.

(2) To calculate $n_1 D_1 = \int_0^{2\pi} \rho \cos 2\theta d\theta.$

We have $\rho = \frac{n_1 h^2}{\pi} \int_0^\infty e^{-h^2(r^2+v_1^2-2v_1 r\cos\theta)} r dr$

and identically

$$n_1 = \int_0^{2\pi} \rho d\theta = \frac{n_1 h^2}{\pi} \int_0^{2\pi} d\theta \int_0^\infty e^{-h^2(r^2+v_1^2-2v_1 r\cos\theta)} r dr$$

$$= \frac{n_1 h^2}{\pi} \int_0^\infty e^{-h^2(r^2+v_1^2)} r dr \int_0^{2\pi} e^{z\cos\theta} d\theta . \quad . \quad . \quad . \quad (1)$$

where $z = 2h^2 v_1 r .$

Hence $n_1 \equiv \frac{n_1 h^2}{\pi} \int_0^\infty e^{-h^2(r^2+v_1^2)} r dr . 2\pi J_0(\iota z) . \quad (2)$

But from (1)

$$\int_0^{2\pi} \rho \cos^2\theta d\theta = \frac{n_1 h^2}{\pi} \int_0^\infty e^{-h^2(r^2+v_1^2)} r dr \frac{d^2}{dz^2} \int_0^{2\pi} e^{z\cos\theta} d\theta$$

$$= \frac{n_1 h^2}{\pi} \int_0^\infty e^{-h^2(r^2+v_1^2)} r dr \frac{d^2}{dz^2} \left(2\pi J_0(\iota z)\right)$$

$$= \frac{n_1 h^2}{\pi} \int_0^\infty e^{-h^2(r^2+v_1^2)} r dr \left(2\pi J_0(\iota z) - \frac{1}{z} \frac{d}{dz} \left(2\pi J_0(\iota z)\right)\right)$$

by Bessel's differential equation

$$= n_1 - \frac{n_1 h^2}{\pi} \int_0^\infty e^{-h^2(r^2+v_1^2)} \frac{1}{4h^4 v_1^2} \frac{d}{dr} \left(2\pi J_0(\iota z)\right) dr$$

Integrate by parts:

$$= n_1 - \frac{n_1 h^2}{\pi} \frac{1}{4h^4 v_1^2} \left[e^{-h^2(r^2+v_1^2)} 2\pi J_0(\iota 2h^2 v_1 r) \right]_0^\infty$$

$$+ \frac{n_1 h^2}{\pi} \frac{1}{4h^4 v_1^2} \int_0^\infty dr(-2h^2 r) e^{-h^2(r^2+v_1^2)} 2\pi J_0(\iota z)$$

$$= n_1 + \frac{n_1}{2h^2 v_1^2} e^{-h^2 v_1^2} - \frac{n_1}{2h^2 v_1^2}$$

(the integrated part vanishes for $r = \infty$ and reduces to $\frac{n}{2h^2 v_1^2} e^{-h^2 v_1^2}$

at the lower limit; the part remaining to be integrated is simplified by means of the identity (2)),

$$\text{hence} \quad \int_0^{2\pi} \rho \cos 2\theta d\theta = 2\int_0^{2\pi} \rho \cos^2\theta d\theta - \int_0^{2\pi} \rho d\theta$$

$$= n_1\left(1 - \frac{1 - e^{-h^2 v_1^2}}{h^2 v_1^2}\right).$$

(3) The integral $\int_0^{2\pi} \rho \cos 3\theta d\theta$, and corresponding integrals for any odd multiples of θ, can be found by a simple extension of the method employed for $\int_0^{2\pi} \rho \cos\theta d\theta$. The integrals for even multiples of θ are more troublesome to evaluate, but the method employed for $\int_0^{2\pi} \rho \cos 2\theta d\theta$ always succeeds.

Tables of the two hypergeometrical functions, $F\left(1/6, 5/6, 2, \sin^2\frac{\iota}{2}\right)$ *and* $F\left(-1/6, 7/6, 2, \sin^2\frac{\iota}{2}\right)$, *between the limits iota equals* 90 *and* 180 *degrees.* By C. J. Merfield.

In the method of Mr. R. T. A. Innes for the determination of the secular perturbations,* there are two hypergeometrical functions to be deduced. In an appendix † to this valuable paper, tables of the logarithms of these functions are given with the argument iota for each degree ‡ for the first quadrant.

Tables of these functions facilitate the application of this method in no small degree, and it seemed desirable to extend them, as in many future investigations it will be found that the angle iota will much exceed a quadrant. Taking an example, Eros §—Earth, it will be noted that the modular angle theta, the argument to the tables of elliptical integrals, reaches the value 60°, corresponding to iota 138° 18′, and there will be many other cases in which it exceeds this value.

In the preparation of the tables here given the formulæ || (*l.c.*,

* "Computation of Secular Perturbations," by R. T. A. Innes, *Monthly Notices*, vol. lxvii. 427.

† Tables for the application of Mr. Innes's Method, by Frank Robbins. *l.c.*, 444.

‡ The values of these functions for iota equals 0° have been omitted in the tabulation by Mr. Robbins.

§ "Secular Perturbations of Eros," by C. J. Merfield, *Astr. Nachr.*, 4178–79, Band 175.

|| The values of these functions may be deduced from series. I have given the coefficients of twenty terms, *Astr. Nachr.*, 4215, Band 176, p. 246.

442) of Mr. Innes have been adopted. Seven-figure logarithms have been used, and the *Additions- und Subtractions-Logarithmen* by Zech where necessary, but the powers of the trigonometrical ratios have been formed from the values of these functions tabulated in Vega's ten-figure table, and the results used to seven figures.

Taking

$$\iota_1 = 180 - \iota$$

then $$\cos \frac{\iota_1}{2} = \sin \frac{\iota}{2}$$

and the following relation holds,

$$F\left(-1/6 \ldots \sin^2 \frac{\iota}{2}\right) F\left(1/6 \ldots \sin^2 \frac{\iota_1}{2}\right) +$$

$$F\left(1/6 \ldots \sin^2 \frac{\iota}{2}\right) F\left(-1/6 \ldots \sin^2 \frac{\iota_1}{2}\right) = 216/35\pi.$$

The logarithm of the right-hand member equals 0·29323583 . . . . Writing in this formula the quantities taken from the tables, then it will generally be found that the residual does not exceed a unit in the seventh decimal place of the logarithm in any portion of the tables. This formula might have been used in calculating either function after one of them had been deduced between the limits 90 and 180 degrees, combined with the values tabulated by Mr. Robbins, *loco citato.*

Sydney:
1908 *February* 10.

TABLES.

ι	Log F (1/6 . . . .)	Δ_1	Δ_2	Log F (−1/6 . . . .)	Δ_1	Δ_2
90°	0·0183800	+3991	+41	9·9738258	−5778	−62
91	0·0187832	4032	40	9·9732418	5840	62
92	0·0191904	4072	38	9·9726516	5902	62
93	0·0196014	4110	39	9·9720552	5964	61
94	0·0200163	4149	39	9·9714527	6025	61
95	0·0204351	4188	38	9·9708441	6086	60
96	0·0208577	4226	39	9·9702295	6146	62
97	0·0212842	4265	38	9·9696087	6208	60
98	0·0217145	4303	39	9·9689819	6268	60
99	0·0221487	4342	37	9·9683491	6328	59
100	0·0225866	4379	+37	9·9677104	6387	−60
		+4416			−6447	

r	Log F (1/6 . . . .)	Δ_1	Δ_2	Log F (−1/6 . . . .)	Δ_1	Δ_2
101	0·0230282	+4416	+36	9·9670657	−6447	−58
102	0·0234734	4452	36	9·9664152	6505	58
103	0·0239222	4488	35	9·9657589	6563	57
104	0·0243745	4523	36	9·9650969	6620	58
105	0·0248304	4559	34	9·9644291	6678	56
106	0·0252897	4593	34	9·9637557	6734	56
107	0·0257524	4627	35	9·9630767	6790	55
108	0·0262186	4662	34	9·9623922	6845	56
109	0·0266882	4696	33	9·9617021	6901	54
110	0·0271611	4729	33	9·9610066	6955	53
111	0·0276373	4762	31	9·9603058	7008	53
112	0·0281166	4793	32	9·9595997	7061	52
113	0·0285991	4825	31	9·9588884	7113	52
114	0·0290847	4856	30	9·9581719	7165	50
115	0·0295733	4886	28	9·9574504	7215	48
116	0·0300647	4914	29	9·9567241	7263	49
117	0·0305590	4943	29	9·9559929	7312	48
118	0·0310562	4972	27	9·9552569	7360	45
119	0·0315561	4999	26	9·9545164	7405	46
120	0·0320586	5025	26	9·9537713	7451	44
121	0·0325637	5051	24	9·9530218	7495	44
122	0·0330712	5075	24	9·9522679	7539	41
123	0·0335811	5099	22	9·9515099	7580	41
124	0·0340932	5121	23	9·9507478	7621	39
125	0·0346076	5144	21	9·9499818	7660	37
126	0·0351241	5165	20	9·9492121	7697	36
127	0·0356426	5185	19	9·9484388	7733	34
128	0·0361630	5204	18	9·9476621	7767	33
129	0·0366852	5222	16	9·9468821	7800	30
130	0·0372090	5238	15	9·9460991	7830	29
131	0·0377343	5253	13	9·9453132	7859	26
132	0·0382609	5266	13	9·9445247	7885	25
133	0·0387888	5279	11	9·9437337	7910	24
134	0·0393178	5290	10	9·9429403	7934	20
135	0·0398478	5300	7	9·9421449	7954	17
136	0·0403785	5307	7	9·9413478	7971	15
137	0·0409099	5314	5	9·9405492	7986	13
138	0·0414418	5319	+4	9·9397493	7999	9
139	0·0419741	5323	0	9·9389485	8008	7
140	0·0425064	5323	−1	9·9381470	8015	−3
		+5322			−8018	

ι	Log F (1/6 . . . .)	Δ_1	Δ_2	Log F (−1/6 . . . .)	Δ_1	Δ_2
				9·9373452	−8018	
141	0·0430386	+5322	−4			0
				9·9365434	8018	
142	0·0435704	5318	5			+4
				9·9357420	8014	
143	0·0441017	5313	8			6
				9·9349412	8008	
144	0·0446322	5305	10			11
				9·9341415	7997	
145	0·0451617	5295	12			14
				9·9333432	7983	
146	0·0456900	5283	16			20
				9·9325469	7963	
147	0·0462167	5267	19			25
				9·9317531	7938	
148	0·0467415	5248	21			30
				9·9309623	7908	
149	0·0472642	5227	25			34
				9·9301749	7874	
150	0·0477844	5202	28			40
				9·9293915	7834	
151	0·0483018	5174	31			45
				9·9286126	7789	
152	0·0488161	5143	35			52
				9·9278389	7737	
153	0·0493269	5108	40			55
				9·9270707	7682	
154	0·0498337	5068	43			65
				9·9263090	7617	
155	0·0503362	5025	48			73
				9·9255546	7544	
156	0·0508339	4977	52			79
				9·9248081	7465	
157	0·0513264	4925	58			89
				9·9240705	7376	
158	0·0518131	4867	64			96
				9·9233425	7280	
159	0·0522934	4803	68			107
				9·9226252	7173	
160	0·0527669	4735	76			116
				9·9219195	7057	
161	0·0532328	4659	83			128
				9·9212266	6929	
162	0·0536904	4576	89			137
				9·9205474	6792	
163	0·0541391	4487	98			151
				9·9198833	6641	
164	0·0545780	4389	106			165
				9·9192357	6476	
165	0·0550063	4283	115			180
				9·9186061	6296	
166	0·0554231	4168	126			194
				9·9179959	6102	
167	0·0558273	4042	136			211
				9·9174068	5891	
168	0·0562179	3906	148			231
				9·9168408	5660	
169	0·0565937	3758	163			253
				9·9163001	5407	
170	0·0569532	3595	177			274
				9·9157868	5133	
171	0·0572950	3418	194			300
				9·9153035	4833	
172	0·0576174	3224	213			331
				9·9148533	4502	
173	0·0579185	3011	238			362
				9·9144393	4140	
174	0·0581958	2773	263			403
				9·9140656	3737	
175	0·0584468	2510	294			448
				9·9137367	3289	
176	0·0586684	2216	334			507
				9·9134585	2782	
177	0·0588566	1882	385			581
				9·9132384	2201	
178	0·0590063	1497	462			687
				9·9130870	1514	
179	0·0591098	1035	−607			+890
				9·9130246	−624	
180	0·0591526	+428				

Further Note on the Position of the Sun's Axis of Rotation, as deduced from Greenwich Sun-spot Measures 1874–1885. *Papers of the I.U.S.R. Computing Bureau, No. II.* By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. This note gives for the period 1874–1885 computations similar to those given for the period 1886–1901 in the December No. (p. 98). The discussion is limited to spot-groups which live for ten days. The tables are numbered to correspond with those of the former note.

TABLE I.

Latitude drift between $+65°$ *and* $-65°$ *in each month* (*of the years* 1874–1885).

Latitude.	$-25°$	$-20°$	$-15°$	$-10°$	$0°$	$+10°$	$+15°$	$+20°$	$+25°$
Jan.	+57	+170	−64	−19	−33	+25	00	−21	—	+5
Feb.	+113	−41	−48	−61	+62	−29	+50	−16	+77	+70
Mar.	−27	+145	+15	+45	+16	+30	+80	+97	—	—
Apr.	−55	+100	+79	+23	+34	+55	+113	+61	+118	—
May	−85	—	+12	+39	+45	+5	+62	−22	+10	+5
June	—	−171	+43	+31	−50	+70	−61	+125	+73	+125
July	−23	—	−33	−97	−18	+5	−52	−116	−2	+115
Aug.	—	+15	+35	+28	+17	−34	+27	+23	+57	−5
Sept.	+35	−90	−11	−136	+8	+31	−55	+11	+38	0
Oct.	—	−13	+18	+57	−38	−53	−4	+46	+53	—
Nov.	—	−37	+60	+12	−11	−17	−52	−7	−83	—
Dec.	−75	—	+62	+240	−25	+33	−80	−54	−1	+215
Mean	—	—	+14	+14	+1	+10	+2	+11	—	—

The unit in the tables is $0°{\cdot}01$, and the figures are, in Table I., differences between the mean residual at $+65°$ and $-65°$ in

TABLE II.

Latitude drift between $+35°$ *and* $-35°$ *in each month* (*of the years* 1874-1885).

Latitude.	$-25°$	$-20°$	$-15°$	$-10°$	$0°$	$+10°$	$+15°$	$+20°$	$+25°$
Jan.	+48	+95	+25	+12	−37	+25	−29	−53	—	+15
Feb.	+50	−6	−51	−30	+20	+23	+3	−5	+40	+152
Mar.	+32	+71	−6	+32	+23	+39	+70	−2	—	—
Apr.	−35	+80	+49	+12	−1	+7	+33	+10	+98	—
May	−87	+10	+24	+31	+36	−31	+31	+68	+50	+88
June	+12	−70	+10	+34	+61	+60	−23	+75	+43	+135
July	+51	−88	−1	−16	+25	−10	−10	−38	+12	+87
Aug.	—	+47	+15	−26	+10	−63	+6	+90	+8	+15
Sept.	+2	−6	−24	−87	+26	−4	+3	+8	+41	+12
Oct.	—	+5	+9	+25	+4	−25	−15	+5	+10	—
Nov.	—	−43	+74	+17	−1	−67	−15	+2	−58	—
Dec.	−15	—	+43	+137	+30	+59	−88	−194	+22	+67
Mean	—	—	+14	+12	+16	+1	−3	−3	—	—

longitude from the centre of the disc; and in Table II., similar differences for longitudes $\pm 35^\circ$ from the centre.

Excluding,* at any rate in the first instance, the outside groups (for latitudes above 20°), we can now analyse the remaining 6 columns in each table harmonically, to find an expression of the form

$$A \sin \theta + B \cos \theta + C,$$

where $\theta = 0$ for the middle of January.

Multiplying the coefficients A and B by $\frac{1}{2}$ cosec 65° for Table I., and $\frac{1}{2}$ cosec 35° for Table II., we get the error of the Sun's axis, which we may now express (as Carrington does) in minutes of arc. The separate determinations are given in Table III.

TABLE III.

Quantities representing the Errors of the Sun's adopted axis.

	Latitudes.	−20°	−15°	−10°	0°	+10°	+15°	+20°	Mean.
A from Table I.		−0′·5	−0′·1	+7′·5	+8′·0	+19′·6	+8′·0		+7′·1
A	,, ,, II.	−3·3	+4·3	+3·9	+12·6	+15·5	+12·2		+7·5
B	,, ,, I.	−2·7	+14·3	+1·2	−1·0	+3·9	−2·4		+2·2
B	,, ,, II.	+2·8	+14·5	−8·8	+9·8	−5·9	−32·2		−3·3

Inspection of Table III. suggests the following conclusions:—

(*a*) The accordance between determinations from meridians $\pm 65^\circ$ and $\pm 35^\circ$ is not so marked as before; but the discordances are not systematic. We must remember that the former series was based on 16 years' observations, and the present on less than 11 years'.

(*b*) The mean error of the adopted axis in A is confirmed within very narrow limits, the mean value of A being +7′·3 as against +7′·4 for the period 1886–1901. But the mean value of B for the same period was −7′·8, whereas the measures here discussed give nearly zero. Can we ascribe this difference to accidental error? It is possible that B is more liable to error than A, owing to the fact that its coefficient is large in December and June, and it gets the full effect of any difference between summer and winter. If, for instance, there are fewer good photographs taken at Greenwich during the winter, there will be more Indian used in the series; and any differences between the two would have maximum effect. Such points must be separately examined. Meanwhile, we may remark that the arithmetical sum of the differences between pairs of values of B from Tables I. and II. in the present paper is 66′·1 as against 23′·7 for A; in

* In the former paper two sentences have here been inadvertently transposed. They are repeated here in the correct order.

the former paper these numbers were 20″·7 and 14″·3; so that there is some reason for regarding A as better determined than B, and also for regarding the former results as entitled to greater weight than the present. The best provisional determination of the errors of Carrington's axis, assuming that it remained fixed in space during the period 1874–1901, would therefore be

$$A = +7' \qquad B = -5'$$

giving double weight to the results of the former paper.

(*c*) As regards the discordances between the different zones, we may compare the results of the former paper and the present as follows:—

	A			B		
	−20° to −10°	to +10°	to +20°	−20° to −10°	to +10°	to +20°
1874–1885	+0·1	+8·0	+13·8	+7·2	+0·3	−9·2
1886–1901	+3·2	+5·8	+13·3	−1·3	−13·5	−8·7
Weighted Mean	+2·2	+6·5	+13·5	+1·5	−8·9	−8·9

double weight being given to the second series in taking the mean.

(*d*) We now come to the constant C, the mean value of which is positive (as already remarked in a note on p. 103 of the last paper), instead of negative as before. Rearranging the material in consecutive years, as in the former paper, we get

TABLE IV.

Drift in Latitude between +65° and −65° from central meridian.

Year.	−25°	−20°	−15°	−10°	0°	+10°	+15°	+20°	+25°
1874	—	—	$+54_3$	$+63_3$	-45_4	$+70_6$	-61_4	-245_2	—	—
1875	—	—	—	$+75_5$	$+87_4$	-34_5	0_1	$+67_2$	—	—
1876	—	—	—	-112_4	$+55_2$	—	$+57_2$	—	—	—
1877	—	—	—	$+20_1$	—	-6_5	$+90_1$	—	—	—
1878	—	—	—	—	—	$+46_2$	$+5_1$	—	—	—
1879	—	$+45_1$	—	—	—	—	—	—	—	-80_1
1880	$+31_3$	-171_8	$+19_9$	$+40_1$	—	—	$+100_3$	-4_6	$+491_{12}$	$+13_3$
1881	$+40_2$	$+38_3$	-5_7	$+142_2$	$+203_1$	-35_1	-41_{12}	$+22_9$	$+18_{10}$	$+86_5$
1882	72_2	-14_6	$+82_{10}$	$+64_{12}$	-38_5	$+33_5$	$+11_{12}$	$+21_9$	$+21_7$	—
1883	$+13_2$	-23_8	$+51_6$	$+50_{14}$	-52_{17}	0_7	-51_6	$+37_3$	—	—
1884	-27_1	$+45_1$	$+51_8$	-1_{15}	$+24_{22}$	-12_{22}	$+46_{19}$	-22_3	—	—
1885	—	$+27_3$	-91_5	-191_8	$+251_5$	$+101_2$	$+51_4$	-27_2	—	—

In Table V. the results of the former paper are repeated, so as to show the annual results for the whole series of years.

TABLE V.

Year.	Mean Drift.	No. Spots.	Year.	Mean Drift.	No. Spots.
1874	−7	22	1888	−90	10
1875	+40	17	1889	−34	10
1876	−28	8	1890	−3	6
1877	+11	7	1891	−2	57
1878	+32	3	1892	−30	107
1879	−18	2	1893	−19	135
1880	+17	40	1894	−24	118
1881	+18	52	1895	−40	94
1882	+27	68	1896	−39	49
1883	−4	83	1897	−39	44
1884	+16	91	1898	−47	32
1885	+2	79	1899	+21	14
1886	−4	37	1900	−7	9
1887	0	18	1901	+50	3

The unit of the table is $0^{\circ}\cdot01$, and the drift tabulated takes place in 130° of longitude. The figures have not yet been corrected for error of the Sun's axis, so that the results for years with few spots are subject to sensible corrections, depending on the particular months in which the spots were visible.

But the main feature of the table seems to be an oscillation from a drift of about $+20$, or $+33'$ per rotation, in 1880 to an opposite drift of -25, or $-40'$ per rotation in about 1893.

Now one possible explanation of these figures is that the Sun's axis is not stationary, but is describing a cone in space, in a period which may be one of several.

(α) If the period were about 26 years, the effect would be reversed in 13 years, but it is easily seen that the angular radius of the cone would have to be too large to fit the facts.

(β) If the period were nearly a submultiple of a year, say six months, Tables I. and II. would show it. Analysis does not yield a suitable result.

(γ) It seems most likely that the period is nearly a year. If it were exactly a year, and we had adopted a wrong axis so as to get an apparent drift in latitude, the error and its effects would travel round with the Earth, so that we should get a spurious constant drift. If the period differs from a year, the spurious drift would slowly change, going through a complete cycle in the L.C.M. of the period and the year. For instance, if the cycle is about 26 years, the period of the wobble of the Sun's axis must differ from one year by about half a month.

(δ) The drift may, however, be a physical drift on the surface of the Sun; and we can distinguish between these rival hypotheses by examining spots which return after disappearing. If the drift is physical, it will persist on the side of the Sun remote from the Earth; if due to the rotation of the axis, it will be reversed on the other side. Since the amount per rotation is large near a maximum (say 40′ per rotation), even a cursory examination should settle the point. Carrington's observations show no tendency to physical drift, as may easily be seen from the results 130 to 213, or the table pp. 213–219, or the summary on p. 220. Nor does an examination of the spot returns tabulated by Father Corte (*Mon. Not.*, vol. lx.) give much support to the hypothesis of a physical drift.

On the whole, hypothesis (γ) seems best worthy of further study.

But before deducing definitive elements for the axis, we await the results deduced from the measures of C. H. F. Peters, and recently published by Professor Frost. These are nearly completed, and can probably be given in the next number of the *Monthly Notices*.

Magnitude of η Argus, 1908. By R. T. A. Innes.

Comparison star used, Gilliss 1332 of 7·6 mag., yellowish or reddish, 4 on Chandler's colour-scale. This star is 8′ N.pr. η Argus.

1908.	Mag.	Colour.
Apr. 2	7·8	5
7	7·8	
13	7·7	6
June 2	7·7	6
Mean	7·75	5·7

A low-power field, with both stars in view, on the 9-inch refractor.

Johannesburg:
1908 *June* 3.

Photometric Measurements of Neptune, January to April 1908. By J. M. Baldwin, M.A., 1851 Exhibition Scholar (Melbourne).

(*Communicated by Sir David Gill, K.C.B., F.R.S.*)

1. The observations which are recorded in this paper were undertaken chiefly to ascertain whether it were possible to recognise a short-period variation in the brightness of Neptune such as that which Maxwell Hall believed he had found in 1883 and again in 1884.

The first announcement of a variation was made in the *Monthly Notices* (xliv. 257, 1884). The observations made by Mr. Hall were *estimations*, generally in fourths of a magnitude, of the difference between Neptune and a star in the same field, assumed as of magnitude 8·5 (B.D. + 15°, 453, mag. 8·5). On the night of 1883 November 29 Neptune seemed to him fainter than on the two preceding evenings, being "as nearly as possible" of the same brightness as the comparison star, but later in the evening Neptune appeared somewhat the brighter. From then till December 14 the estimated magnitude varied between 7·5 and 8·5, but during the nights of December 15, 16, and 18 no variation could be observed. From his observations he deduced a period of 7·92 hours. On December 27 and again on 1884 January 8 Neptune was compared with another star (B.D. 15°, 446, mag. 7·5), but "no variation was noticed on either night, yet Neptune was undoubtedly brighter on January 8 than on December 27" (*Observatory*, vii. 73, 1884).

He observed Neptune again at the next opposition, and in a short note in the *Observatory* (viii. 26, 1885) says: "Careful photometric measures gave November $29^{d}\ 9^{h}\ 44^{m}$ G.M.T. as an epoch of maximum brilliancy, but the variation was only 0·4 magnitude between max. and min. The period of $7^{h}{\cdot}92$ observed last year has been fully confirmed." I have unfortunately been unable to find any information as to the observations beyond that contained in this brief paragraph.

2. A considerable number of *measurements* of Neptune with the meridian photometer have been made at Harvard College Observatory,* almost all by E. C. Pickering, λ Ursæ Minoris

* *Observatory*, vii. 134, 1884, and viii. 111, 1885; *Harvard Annals*, xxiv. p. 265 and xlvi., part ii., p. 203. In this paper the early comparisons, 4 in number, of Neptune with Uranus, made by Zöllner (*Photometrische Untersuchungen*, p. 150) and the 8 Harvard comparisons with α Arietis in 1878 (*Harvard Annals*, xl. p. 224) have not been made use of.

being used as standard. The number of observations and results obtained are given in Table I.

Table I.

Year.	Observations.	Nights.	Mean Mag.	Mean Error.
1882–83	11	11	7·71	±·11
1883–84	5	5	7·77	±·09
1884–85	9	9	7·63	±·11
1894	5	5	7·71	±·13
1895–96	5	5	7·50	±·10
1896	5	5	7·63	±·02
1897–98	16	10	7·63	±·10

3. The observations of Müller * at Potsdam are more numerous. These are *measurements* with the Zöllner photometer of the difference in magnitude between Neptune and a suitable comparison star or stars, the same throughout any one opposition. The mean results, etc. for the separate years will be found later on in this paper (Table IV). The mean of all the 138 observations is 7·66 on the scale used in Band viii.; this is exactly the same as the mean of Pickering's 56 observations between 1882 and 1898.

4. In the observations given below, a Zöllner photometer attached to the Steinheil refractor (13·5 cm. aperture and 216 cm. focal length, called photometer D in the *Potsdam Publications*) was used throughout. Two comparison stars were chosen near Neptune, namely B.D. +24°, 1457 and +22°, 1531, the magnitudes given in the Potsdam General Catalogue being respectively 7·97 and 7·18. As usual, four readings were taken for each object, one in each quadrant of the intensity circle (which was always turned in the same sense), the star being respectively to the left of, above, to the right of, below, the artificial star. The complete observation consisted in observing first one comparison star, then Neptune, then the other comparison star. For the next observation the stars were taken in the reverse order, and so on throughout the whole series. The complete results are given in Table II.

* *A.N.*, 2600, Bd. cix. 121, 1884, or *Observatory*, vii. 264, 1884; *Pots. Publ.*, Bd. viii. It is to be noticed that the magnitudes as given in the *A.N.* are not referred to the same scale as those in the collected results in the *Pots. Publ.*

[Table

TABLE II.

1908.	G.M.T. d	h	m	Z.D. °	M_1.	M_2.	Mean.	Reduction.	M_0.	Phase. °	Residual.
Jan.	25	6	8	51·7	7·85	7·98	7·92	+ ·01	7·93	0·68	− ·06
		7	6	43·5	8·00	7·93	7·97		7·98		− ·01
		9	46	30·4	8·02	7·85	7·94		7·95		− ·04
Feb.	3	10	12	32·5	8·04	7·89	7·96		7·97	0·98	− ·02
		11	3	37·1	7·80	7·93	7·86	+ ·01	7·87		− ·12
Feb.	9	11	18	41·9	7·96	7·96	7·96	·00	7·96	1·15	− ·03
	10	7	24	34·0	7·93	7·91	7·92		7·92	1·17	− ·07
		8	5	31·3	7·88	8·01	7·95		7·95		− ·04
		9	43	32·5	7·97	7·99	7·98		7·98		− ·01
	14	7	22	32·9	7·93	8·06	7·99	·00	7·99	1·27	·00
Feb.	19	8	42	30·5		8·04	8·04	− ·01	8·03	1·38	+ ·04
Mar.	2	9	52	41·9	8·10	8·11	8·11	− ·02	8·09	1·63	+ ·10
		10	44	49·3	8·17	8·13	8·15		8·13		+ ·14
		11	10	53·1	8·06	8·02	8·04	− ·02	8·02		+ ·03
	16	9	7	43·4	8·23	8·12	8·17	− ·03	8·14	1·82	+ ·15
Mar.	16	9	19	45·2	7·91	8·00	7·95	− ·03	7·92		− ·07
	22	9	21	48·7	8·01	8·07	8·04	· ·04	8·00	1·87	+ ·01
		9	55	53·7	8·08	8·09	8·08		8·04		+ ·05
	24	9	47	53·7	8·14	7·99	8·06		8·02	1·89	+ ·03
		10	16	58·1	8·07	8·07	8·07		8·03		+ ·04
Mar.	24	10	47	62·8	7·99	8·04	8·01		7·97		− ·02
	25	9	2	47·7	8·01	8·02	8·01		7·97		− ·02
		10	25	60·1	8·05	7·98	8·01		7·97		− ·02
		10	48	63·6	8·09	8·06	8·07	− ·04	8·03	1·89	+ ·04
	26	9	0	48·0	8·02	8·02	8·02	− ·05	7·97	1·90	− ·02
Mar.	26	10	21	60·1	7·94	8·03	7·99		7·94		− ·05
		10	48	64·2	8·00	8·10	8·05		8·00		+ ·01
	27	8	31	44·5	7·95	8·03	7·99		7·94		− ·05
		9	19	51·3	8·09	8·07	8·08	− ·05	8·03	1·90	+ ·04
Apr.	26	8	28	61·3	8·22	8·06	8·14	− ·08	8·06	1·76	+ ·07
Apr.	28	7	39	54·9	7·98	8·00	7·99		7·91	1·74	− ·08
		8	19	61·0	8·15	8·07	8·11	− ·08	8·03		+ ·04
								Mean	7·990		± ·059

Jan. 25. Slightly foggy.
Feb. 14. Hazy, but no clouds visible in bright moonlight.
Feb. 19. Clouded over, weight ½.
Mar. 2. Not very clear, hazy near horizon.
Mar. 16. Sky bad, and so observations not reliable, weight ½.
Apr. 26. Cloud slowly rising, but observations probably good.

In this table the first two columns give the date and G.M.T. of observation, and the corresponding zenith distance of Neptune, the next columns the observed magnitudes (corrected for atmospheric absorption) referred to the brighter and fainter comparison stars respectively, and the mean. Each observation gave the difference in magnitude between the comparison stars; from the whole series the mean was 0·83 mag., and so the values for the comparison stars are taken as 7·16 and 7·99 respectively. In the sixth and seventh columns are given the correction to mean opposition and the magnitude in mean opposition. The eighth column gives the phase angle; this remains always so small that it is not to be expected that any variation of magnitude with phase angle can be recognised with certainty. The last column gives the difference, observed mag. – mean magnitude.

A glance at the column of residuals shows that the agreement between the observations is extremely satisfactory, the mean error of one observation, giving a weight $\frac{1}{2}$ to certain observations as indicated, being only $\pm$ ·059 mag.

5. Amongst the foregoing material there are three series of observations which are especially suited to detect a short period variation in brightness such as that announced by Hall, the epoch and amplitude of which, owing to movement of the markings supposed to give rise to the variation, may be gradually changing.

(1) Müller's 72 observations on 37 evenings between 1884 Aug. 24 and 1885 March 10.

(2) Pickering's 16 observations on 10 evenings between 1897 Dec. 11 and 1898 March 3.

(3) The above 32 observations on 15 evenings.

If these observations be examined it will be at once evident that they show no trace of a short-period variation, the residuals being just such as occur in photometric work. However, as a direct test, the series have been taken separately, the phase computed for a period 7·92 hours and an arbitrary epoch, and the light curve plotted. In the case of Müller's longer series, this has been done also for parts of the series. The result is in every case the same; the observations furnish no evidence whatever of a variation of period 7·92 hours. In particular, Müller's observations from 1885 Jan. 8 to Feb. 24 and those given in this paper are especially conclusive on this point, as the observations are well distributed over the curve. With regard to his observations from 1884 Aug. 24 to 1885 March 10, Müller says (*Pots. Publ.*, viii. p. 352), "during the whole period of nearly 7 months not the slightest trace of a periodic variation in light can be recognised. The magnitude in mean opposition varies between 7·39 and 7·81, and such a difference can be very well explained by uncertainty in the measurements if the fact that these extreme values were obtained when the planet was very low, and under unfavourable conditions, be taken into account"; and a little further on, "the observations of the following years also, although not nearly so numerous, are in direct opposition to the hypothesis of a periodic variation in the light from Neptune."

This latter remark may be extended to the whole of the photometric observations of Neptune.

6. Looking now at the evidence which Hall gave as showing a periodic variation of brightness, it consists of—

(1) A series of 25 estimations of difference of magnitude.

(2) Some photometric observations, of which particulars are not available.

If the first set of observations be plotted to obtain the light curve, it will be found that the assumption of variation practically rests on 3 observations, for, apart from these, no residual occurs which could not be explained by the method of observing, the greatest being 0·3 mag. from a mean 7·8. Of these three, two are the early observations of Nov. 29 (mag. 8·5) and Dec. 1 (mag. 8·3), and it was not until after the first observation that a diagonal prism was used to alter the relative positions of Neptune and the comparison star; and the third, on Dec. 7 (mag. 8·3), requires in any case a correction of more than −0·4 to bring it on to a curve through the other observations.

From the second set a variation of 0·4 mag., occurring about 1885 Nov. 29, was deduced. In the absence of details it is impossible to say what instrument was used, how many observations were made, or over what period they extend. Against the hypothesis of variation, however, there is very conclusive evidence in Müller's observations before and after this epoch, for there are continuous observations from Oct. 23 to Nov. 19, and again from Jan. 8 to Feb. 24, which effectively include any assumption of variation during these periods, and observations on Dec. 1, 2, and 16, which give no evidence in favour of it.

It would appear, then, in the highest degree probable that the "variation" announced is due to errors of observation, especially when account is taken of the small amplitude deduced.

7. The mean value of the brightness of Neptune in mean opposition, as determined by the present series of observations, is 7·99, the mean error of one observation being ±·059, the system to which the magnitude is referred being that of the Potsdam *Durchmusterung*.

The comparison stars used by Müller, the magnitude on the system of his Potsdam planetary observations, and the magnitude from the Potsdam General Catalogue are given in Table III.

TABLE III.

Comparison Star.	B.D.	Mag.	Mag. P.G.C.	Reduction.
52	+13°, 411	7·25	7·52	+·27
43	+15°, 430	6·29	6·68	+·39
34	− 4°, 4376	5·41	5·54	+·13
49	+16°, 458	6·95	7·34	+·39
56*	+16°, 432	7·96	8·23	+·27

* Owing to a clerical error, this comparison star has been printed as 55 instead of 56 throughout the observations of Neptune, and the discussion of the result of these in vol. viii. of the *Potsdam Publications*. Beyond writing 56 for 55 no change requires to be made.

Stars 34 and 56 are not contained in the General Catalogue, but from *Pots. Publ.*, Band viii. p. 233,

34 – β Herculis is 2·53 mag.
and 56 – σ Arietis is 2·51 mag.

Also from the General Catalogue,

β Herculis is 3·01 mag.
and σ Arietis is 5·72 mag.,

and hence the magnitudes given in Table III are obtained.

Next, combining these results with the table given on p. 354 of Müller's paper we get Table IV.

TABLE IV.

Year.	Number of Observation.	Mag.	Prob. error of one Observation.	Comparison Star.	Reduction.	Reduced Magnitude.
1878	11	7·70	±0·051	52	+·27	7·97
1881–2	17	7·75	·071	52	+·27	8·02
1883	6	7·75	·095	43	+·39	8·14
1883	7	7·70	·085	34	+·13	7·83
1884–5	72	7·62	·056	49 and 56	+·33	7·95
1885–6	15	7·69	·074	49 and 56	+·33	8·02
1886–7	7	7·62	·045	49 and 56	+·33	7·95

The mean magnitude on the system of the Potsdam *Durchmusterung* is thus 7·97.

For the comparison with Pickering's observations, the difference Potsdam – Pickering is given on p. 34, of the introduction to the Potsdam General Catalogue, as follows:—

Magnitude.	W	GW
7·50–7·99	+0·36	+0·29
8·00–8·49	+0·37	+0·30

If the colour of Neptune be assumed as between W and GW the reduction +0·34 cannot be far from the truth, and this gives 8·00 as the reduced value of Pickering's mean result.

It is seen, then, that the results of the present series are in very close agreement with the previous observations of Müller and Pickering.

8. The results arrived at from the present series of observations in conjunction with previous observations of Neptune may be summarised as follows:—

(1) The variations of the estimated brightness of Neptune announced by Hall are probably due to errors of observation, as the measurements of other observers about the same time, and also

at other oppositions, give not the slightest trace of any such variation. Consequently the time of rotation of 7·92 hours deduced by him must be regarded as unsupported by observation.

(2) The observed magnitude of Neptune in mean opposition is 7·99 on the system of the Potsdam *Durchmusterung*, showing, in extremely satisfactory agreement with Müller's value 7·97, and Pickering's value 8·00.

In conclusion, I wish to record my thanks to Professor Müller, at whose suggestion these observations were made, for the interest which he has shown throughout this work.

Astrophysical Observatory, Potsdam:
1908 *June* 6.

Observations of helium D_3 absorption in the neighbourhood of Sun-spots in 1907. By Captain R. A. C. Daunt, D.S.O.

The following observations of D_3 absorption in the neighbourhood of sun-spots were made with a 3-inch equatorial refractor and a large Thorp, prism-grating spectroscope, 15,000 lines to the inch, the eyepiece ordinarily employed being a Steinheil monocentric 20 m/m power 52. The identification of the several groups according to the Greenwich numeration will, it is hoped, be of service for comparison with spectro-heliographic and cognate studies of the same groups. In Table I. the numbers of the various groups and the dates on which they were observed are given, as well as the type or phase in the life-history of the groups at the times of observation, according to the classification of Father Cortie in his paper "On the Types of Sun-spot Disturbances," *Astrophysical Journal*, xiii., 4th May 1901.

In all the cases of absorption of D_3 observed, the dark line was never seen over the umbræ or penumbræ of the spots of any group, but between the spots, or on the photosphere in their neighbourhood. It would appear that some of these groups showed D_3 in absorption on certain days only, and not on others. This may be real; but as the appearance of D_3 is very intermittent, it is more likely to be due to the hour of observation; and had I been able to keep the groups under observation throughout the day, D_3 would probably have been seen near the spots some time or other of the day during the apparition, provided the groups remained active and agitated.

In Table II. are given the Greenwich numbers of those groups in the neighbourhood of which D_3 was not observed. For the reason stated above, some of these groups may have shown the absorption effect of D_3 at some time, but it was not observed.

As, however, most of these spots were small, or of a regular and quiescent nature, it is perhaps unlikely that D_3 was reversed.

A table follows showing the proportion of the total number of fresh groups observed each month with those that showed D_3.

There would appear to be a discrepancy here, and the proportions as shown by the Greenwich numbers appear to be higher. This is due to a different system of numbering. For instance, the group consisting of the Greenwich Nos. 6075, 6076, 6081, 6085 has for the purposes of the table been counted as one group. This is, I think, under the circumstances, more accurate, as by the Greenwich numbers it would appear that four different groups showed D_3 in absorption, whereas these spots were obviously part of one disturbance, and should only be counted as one disturbance for this purpose.

TABLE I.

Group in which D_3 was observed as a dark line.

Greenwich No. of Group.	Dates of Observation.	Type of the Group.
	1907	
6059	Jan. 2, 3	IV_b
* 66	,, 3	IV
67	,, 3	III
70	,, 3	I
75		
* 76	,, 16	III_b
81		
85		
94	,, 24	III_a
6099	,, 30, Feb. 4	III_a, IV_b
6100	,, 30, ,,	III_a, IV_b
* 03	Feb. 4, 5	III, IV_b
04	,, 4, 5, 11	I, III_b, III_a
* 08	,, 9, 11, 13, 16, 17	V, III_a, III_a, V, V
* 36	Mar. 3, 4	IV_b, IV_a
37	,, 4	III
39	,, 5, 6	IV_b, IV_b, IV_a
* 40	,, 5, 6, 14, 17	IV_b, IV_a
50	,, 22	I
55	,, 23	IV_e
* 56	,, 30	I
58	Apr. 4	III, IV
59	Mar. 31, Apr. 2, 4, 9	IV, III_a, III_a, IV_b
68	Apr. 15, 16, 17	I, I, II
72	,, 21, 23, 29	III, IV_b, IV_a
75	,, 26, 27	I, I
81	May 4, 5, 6	III, III, IV_a
* 84	,, 4, 5, 6, 14	IV_e, IV_e, IV_e, IV_e
6185	,, 13, 14	III_a, III

TABLE I.—*continued.*

Greenwich No. of Group.	Dates of Observation.	Type of the Group.
	1907	
6187	May 14, 16, 17	I, II_c, III
6188	,, 14, 16	III, II
6230	Aug. 5	I
31	,, 5	I
36	,, 18, 23, 24	III, IV_b, IV_b
* 38	,, 24	IV_a
39	,, 23	IV_b
* 41	,, 30, Sept. 6	III_a, III_a
43	Sept. 6	IV_b
45	,, ,,	I
47	,, ,,	III
55	,, 20, 21	I, I
56	,, 22	IV_a
59	,, 20, 21, 22, 25, 26, 27, 29	III_b . . . . III_a
60	,, 20, 21, 22, 26	IV_a, IV_d, IV_d, IV_d
62	Sept. 20, 21	IV_a, IV_d
65	,, 25, 26, 27, 29	III_a . . . . III_a
68	,, 27, 29	I, IV_b
70	,, 29	I
75	Oct. 9	I
76	,, 9, 13, 14, 16	I, III . . . . III_a
77	,, 9, 13, 14, 16	I, III_a . . . . III_a
78	,, 13, 14	III_a, III_a
79	,, 9, 14, 16	I, IV_c, IV_b
81	,, 14, 16	IV_c, IV_b
82	,, 14, 16, 19, 23, 24	IV_b, IV_c, IV_b, IV_b, IV_b
* 83	,, 14, 16, 19, 23, 24, 26, 27	III_a . . . . IV_e, IV_c
* 84	,, 16, 19, 23, 24, 26, 27	III_a . . . . IV_e, IV_c
85	,, 19, 23, 24, 26, 27	III_a . . . . IV_e, IV_c
* 89	,, 23, 26, 27, Nov. 1	IV_a, IV_b, IV_b, I
91	Nov. 1	II_c
6292	,, ,,	I
*6316	Dec. 2, 3	IV_b, IV_b
18	,, 6, 7, 9	IV_a, IV_a, IV_a
* 19	,, 6, 7, 9, 12, 14	IV_b V
20	,, 9	III
* 21	,, 9, 12	IV_b, II_a
23	,, 14, 18	I, II_c
6324	,, 18	IV

An asterisk denotes that the group so marked was the return of a former group. The appearance of the absorption-line in the cases observed was lumpy, patchy, broken, and thickened, sometimes distorted towards the violet, as in groups 6259, 65, 68, and once, in groups 6283, 84, distorted towards the red. Group 6137, March 4th, 10·30 G.M.T., furnished the most remarkable appearances of D_3 absorption observed since those of the big spot of July 1906. The appearance of dark D_3 as a fine continuous dark line has not been recorded.

In every case of observation of reversal of D_3 the C line of hydrogen was sympathetically affected, being twisted and lumpy, distorted towards the red and the violet, accompanied by brilliant reversals, sometimes over considerable areas in the neighbourhood of spots, as in the groups 6283, 84, 85, Oct. 21, when, too, the sodium lines D_1 and D_2 were reversed.

Magnificent reversals were seen in group 6103, Feb. 5th, when a large prominence arising amongst scattered spots could be traced across the disc and out beyond the limb. In group 6188 the line was thick, with black lumps and bright reversals in the faculæ surrounding the group, and in group 6247 violently disturbed, greatly distorted to the red and violet, and very lumpy, with brilliant reversals.

TABLE II.

List of Spot-groups in which the D_3 line was not observed.

Greenwich No.	Type.	Greenwich No.	Type.
6061	I	*6113	IV, I
62	I	* 15	I
68	I	* 17	IV
* 74	I	18	III
77	IV_a	19	I
79	I	20	I
* 86	I	21	I
* 87	IV_a, I	* 23	IV
88	IV_b	27	IV, I
* 90	IV	28	I
93	I	29	I, III
97	I	31	I
*6098	IV, I	33	I
6101	IV, I	34	IV
02	I	35	I
* 05	I	38	I
07	IV_a	45	I
09	I	46	I
11	I	52	I
*6112	I	*6153	I

TABLE II.—*continued.*

List of Spot-groups in which the D_3 line was not observed.

Greenwich No.	Type.	Greenwich No.	Type.
*6154	I	*6246	I
* 61	IV	48	I
64	IV, I	61	I
65	I	63	IV, I
70	I	64	I
74	I	* 67	IV
*6189	IV	69	I
6220	IV	71	IV
23	I	72	I
25	I	* 73	IV
26	IV	* 80	IV
27	I	*6288	I
29	I	6317	I
32	IV	18	IV
34	IV	22	I
37	I	25	I
6240	I	6326	I

TABLE III.

Showing the proportion of Spot-groups which showed D_3 reversed.

Month.	No. of Days of Observation.	Total No. of Groups observed.	No. of Group in which D_3 Dark.	Percentages.
Jan.	7	21	7	33·3
Feb.	9	18	3	16·7
Mar.	17	21	7	33·3
Apr.	12	13	4	30·8
May	7	6	5	83·3
July	1	4	0	—
Aug.	5	14	6	42·9
Sept.	8	17	9	52·9
Oct.	11	18	7	38·9
Nov.	1	1	1	—
Dec.	9	11	5	43·6
Totals	87	144	54	Percentage on the Totals 37·5

Lynalta, Newtownards,
Co. Down, Ireland.

[*Note added by Father Cortie, S.J.*]

In the above tables I am responsible for the columns headed "Type of the Group," using for the information therein contained, with the kind permission of Captain Daunt, the materials in his observing note-book, the Stonyhurst sun-spot drawings, and the useful monthly "Tables and Notes on Sun-spots" compiled by Mr. Maunder for the *Observatory*. The object of these columns is to ascertain whether the phenomenon of D_3 absorption is confined to, or generally favours, any particular phase in the life-history of sun-spot groups. The more active phases of sun-spot development are represented by Type II. and Type III., and 39·7 per cent. of the groups in which D_3 was observed as a dark line were of these types. In the life-histories of spot-groups it is generally the following part of the group which disappears first, leaving a single spot, with accompanying small companions. This phase, represented by Type IV. with various suffixes, is also largely represented in Table I.; and in his notes on the D_3 reversals, Captain Daunt frequently records that the absorption was observed between the two main spots of a group of Type II. or Type III. form, or in regions following the main spot, that is, in the earlier phases of Type IV. Therefore it follows that in general the phenomenon is indicative of violent local action in the earlier phases of the life-history of groups. However, it is not entirely confined to these phases, as it is recorded in groups 6256 and 6318, examples of round steady spots. But even in these cases, 6256 had a divided nucleus and showed signs of disruptive change, and group 6319 in the neighbourhood of group 6318 was disturbed. The table giving the groups in which D_3 was not observed shows that the spots were in the last stages of their life-history, being in most cases either round steady spots or scattered remnants of groups. But here again there are exceptions, as group 6118 was being developed, and group 6129 passed through an active stage, though on a small scale.

Stonyhurst College Observatory.

Lengths of Axes and Position Angles of 52 Oval Nebulæ.

By Dr. Max Wolf.

Professor Turner has suggested the question whether there is a relation between the apparent form and the situation of oval nebulæ of the type of the Andromeda nebula, viz. Is the ratio of the two axes of the ellipse, visible to us in projection, connected by some rule with the position angle of the major axis?

I have made a first trial with the oval nebulæ of my last catalogue,* putting these together in the following list. The lengths of the two axes a and b are estimated on the plates by a scale in the eye-piece of my microscope; the position angles are estimated with the aid of a cardboard circle at the same.

The visible dimensions of such diffused objects depend in a high degree upon the exposure, the darkness of the plate, the illumination, and the position on the plate, so that the lengths of the axes are affected with some uncertainty. Notwithstanding, there is no doubt that some simple rule, if it exists, must be perceivable from these numbers.

The distribution of the position angles themselves has a maximum at about 30°, and a second fainter at about 160°.

Kgst. Liste 8. No.	N.G.C. No.	α 1875. h m s	N.P.D. 1875. ° ′ ″	Pos. Angle. °	a ″	b ″
111	3935	11 45 55·2	56 54 0	110	45	20
130	new	47 13·9	56 32 0	40	20	15
132	,,	47 15·6	60 3 3	105	45	20
138	,,	47 22·5	55 56 12	130	60	30
154	,,	47 49·7	56 26 35	160	45	12
173	,,	48 51·5	56 9 34	160	20	6
177	,,	48 57·2	56 10 38	80	20	8
193	,,	49 19·4	59 21 47	40	60	15
213	,,	49 53·7	57 52 51	10	60	15
216	,,	49 56·2	57 15 55	135	45	15
232	3986	50 17·2	57 17 0	100	100	20
255	new	51 2·1	58 13 6	165	30	12
257	,,	51 3·8	56 58 12	30	45	10
258	3991	51 5·2	56 57 55	15	15	10
263	3994	51 10·3	57 1 40	10	30	15
271	3995	51 17·2	57 0 37	(?) 30	90	30
303	4020	52 30·8	58 53 23	15	80	20
318	new	52 47·1	58 34 15	140	30	15
346	,,	53 42·6	56 10 0	35	30	12
352	,,	11 53 50·7	58 4 44	45	40	15

* *Königstuhl Nebel-Liste* No. 8, Publ. iii.

Kgst. Liste S. No.	N.G.C. No.	α 1875. h m s	N.P.D. 1875. ° ′ ″	Pos. Angle. °	*a* ″	*b* ″
380	new	11 54 39·0	58 27 28	60	30	20
391	,,	54 52·5	58 1 28	35	30	6
444	,,	55 46·5	58 21 11	15	20	6
452	,,	55 58·1	59 26 30	70	75	15
455	,,	56 1·2	59 50 6	60	45	20
483	,,	56 57·4	59 53 1	55	90	20
498	,,	57 25·8	59 35 23	150	60	15
502	,,	57 32·6	58 11 44	10	45	15
510	4062	57 40·2	57 24 25	100	180	60
529	new	58 10·4	58 36 47	30	30	8
541	,,	58 21·9	58 8 41	160	20	12
545	,,	58 24·4	56 48 29	5	45	8
548	,,	58 26·4	57 2 30	30	45	(?) 5
582	,,	59 26·2	59 14 18	60	30	15
584	,,	59 28·1	58 15 27	170	20	8
593	,,	59 34·9	57 57 22	45	22	7
607	,,	11 59 44·9	56 24 27	120	30	12
680	,,	12 0 45·0	58 14 36	30	30	15
682	,,	0 46·4	56 18 28	70	40	15
699	,,	1 12·4	56 28 13	110	30	6
711	? 4122	1 39·8	56 21 53	160	20	12
715	new	1 46·3	55 47 52	5	30	15
719	,,	1 58·2	56 42 39	175	30	12
725	,,	2 21·3	55 40 27	40	30	15
726	,,	2 21·8	57 7 59	80	30	15
737	4132	2 39·6	60 3 4	30	45	20
740	new	2 44·4	54 59 56	175	30	12
741	4134	2 47·8	60 7 39	150	90	45
743	new	2 50·4	58 23 47	25	75	20
764	4150	4 12·8	58 54 8	150	90	45
769	new	4 57·2	56 42 35	65	45	30
770	,,	12 5 44·4	56 34 10	150	45	30

Astrophys. Observatory, Königstuhl, Heidelberg,
1900 *July* 2.

[By the courtesy of the secretaries, the above very interesting measures have been included in the present number, but there was not time to prepare any discussion of them. A very rough

preliminary analysis indicates that the measures lend some support to the hypothesis of a systematic orientation of axes parallel to the Galaxy (see *M.N.*, lxvii. p. 333 and p. 498), but a complete discussion must be deferred.—H. H. Turner.]

Erratum.

In *Monthly Notices*, vol. lxvii. p. 321 (Baxendell's Observations of U Geminorum),

for 1858 Nov. 14 *read* 1858 Nov. 15 (as on p. 324).